Fields and Waves

Lesson 4.4

TRANSMISSION LINES - INPUT IMPEDANCE

Darryl Michael/GE CRD

Input Impedance

Do Experiment 1

- After Experiment: Cable+Load can look very different than Load

- Characterize V/I as Impedance

Input Impedance

From previous class:

$$\hat{V}(z) = V^+(z) + V^-(z) = V_m^+ \cdot e^{-j\cdot\beta\cdot z} \cdot (1 + \Gamma(z))$$

What about I?

$$\hat{I}(z) = \frac{V^+(z)}{R_c} - \frac{V^-(z)}{R_c} = \frac{V_m^+}{R_c} \cdot e^{-j\cdot\beta\cdot z} \cdot (1 - \Gamma(z))$$

Also,

$$\Gamma(z) = \Gamma_L \cdot e^{-2\cdot j\cdot\beta\cdot(L-z)}$$

Input Impedance

Form the Ratio:

$$\frac{\hat{V}(z)}{\hat{I}(z)} = R_c \cdot \frac{1+\Gamma(z)}{1-\Gamma(z)}$$

Book calls this $Z_{in}(z)$

We are primarily interested in z=0 value

- treat connection to rest of circuit as 2 port with,

$$Z_{in}(z=0) = R_c \cdot \frac{1+\Gamma(z=0)}{1-\Gamma(z=0)}$$

Input Impedance

After lots of algebra, one can show:

$$Z_{in}(z=0) = R_c \cdot \frac{Z_L + j \cdot R_c \cdot tan(\beta \cdot L)}{R_c + j \cdot Z_L \cdot tan(\beta \cdot L)}$$

Special Case example: Z_L=0 (short circuit)

$$Z_{in}(z=0) = R_c \cdot \frac{0 + j \cdot R_c \cdot tan(\beta \cdot L)}{R_c + j \cdot 0 \cdot tan(\beta \cdot L)} = j \cdot R_c \cdot tan(\beta \cdot L)$$

Input Impedance - SHORT CIRCUIT

$$Z_{in}(z=0) = j \cdot R_c \cdot \tan(\beta \cdot L)$$

Can change Z_{in} by changing these two parameters

- Fix β, vary L - different effects
- Vary β, fix L - get same effects

Note that L is the length of the Transmission Line

Input Impedance - TL

Setup for Problem 1a & 1b

1a. Open Circuit Case

$$Z_L = \infty$$

$$Z_L = R_c \frac{Z_L + j \cdot R_c \cdot tan(\beta \cdot L)}{R_c + j \cdot Z_L \cdot tan(\beta \cdot L)}$$

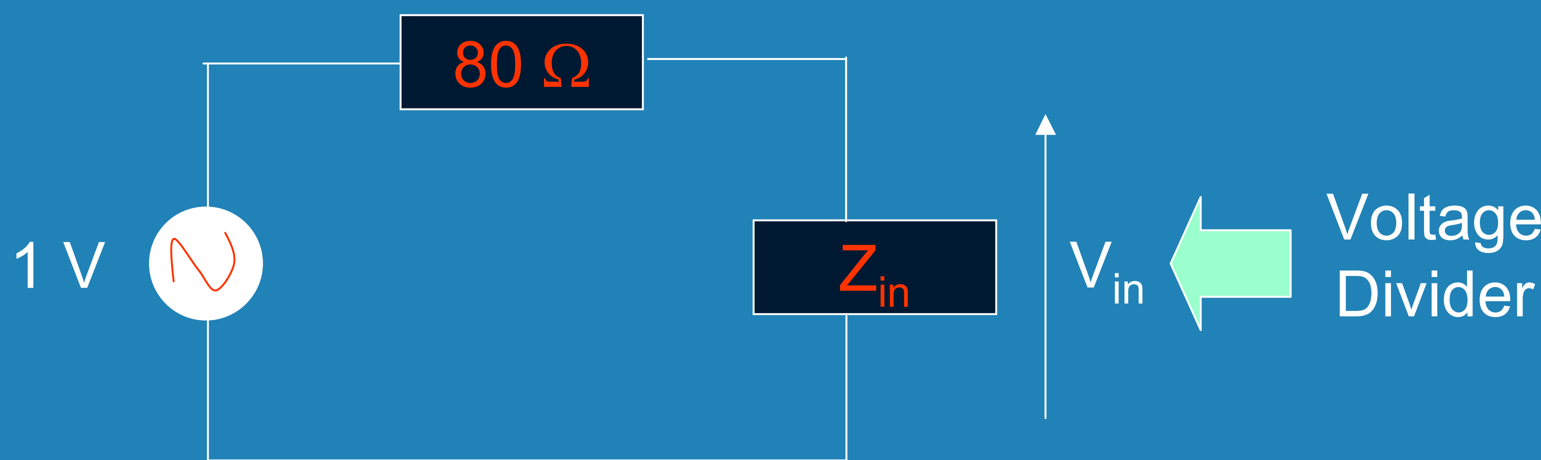
small

small

1b. Z_L *= 93Ω - lots of complex algebra*

Problem 1a and 1b

We know Z_{in} (z=0) - treat as 2-PORT



$$Power = \frac{1}{2} Re\{V_{in} \times I_{in}^*\} = \frac{1}{2} Re\left\{\frac{V_{in} \times V_{in}^*}{Z_{in}^*}\right\} = \frac{1}{2} Re\left\{\frac{|V_{in}|^2}{Z_{in}^*}\right\}$$

Do problem 1b - parts 2 &3

Problem 1a and 1b

In a Lossless Transmission Line, P_{in} flows into the Transmission Line and it is dissipated at the LOAD

$$P_{in} = \frac{1}{2} \cdot \frac{|V_L|^2}{Z_L}$$

Do Problem 1b - part 4

$$\hat{V}(z) = V_m^+ \cdot e^{-j \cdot \beta \cdot z} \cdot (1 + \Gamma(z))$$

$$\hat{V}(z=0) = V_{in} = V_m^+ \cdot e^{-j \cdot \beta \cdot z} \cdot (1 + \Gamma(z=0))$$

$$\Rightarrow V_m^+ = \frac{V_{in}}{1 + \Gamma(0)}$$

Can then plug back and get the full phasor

Do Problem 1b - part 5