Gradient, Line integrals, & Curl

Reading assignment
Ulaby, 3-4
Connor and Salon, II-39 → II-44

Software
div_curl_example.m
Maple (check your solutions)

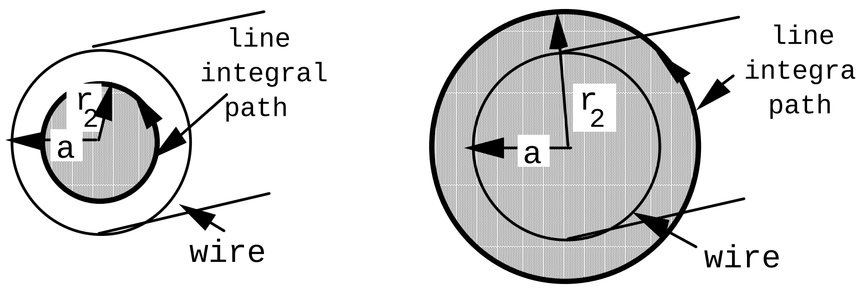
Problem 1 - Line integrals & curl
The magnetic field of a straight wire of radius a which has a constant current density J_0, is given by:

$\mathbf{B} = \mu_0 J_0 r / 2\, \mathbf{a}_\varphi$ inside the wire ($r < a$)

$\mathbf{B} = \mu_0 J_0 a^2 / (2 r)\, \mathbf{a}_\varphi$ outside the wire ($r > a$).

where μ_0 and J_0 are constants.

a. Calculate $\oint \mathbf{B} \bullet \mathbf{dl}$ around the 2 paths shown in the figure below. (The drawing shows a cross-sectional view as if the wire had been cut).



b. Calculate $\nabla \times \mathbf{B}$ for <u>both</u> regions.

Problem 2 - Properties of fields with curl
The electric field created by a cylinder of radius a with constant charge density ρ_0 is:

$\mathbf{E} = \rho_0 r / (2 \varepsilon_0)\, \mathbf{a}_r$ inside the cylinder ($r < a$) and

$\mathbf{E} = \rho_0 a^2 / (2 \varepsilon_0 r)\, \mathbf{a}_r$ outside the cylinder ($r > a$).

where ρ_0 and ε_0 are constants.

a. Verify that $\oint \mathbf{E} \bullet \mathbf{dl} = 0$ on the same paths as above and that $\nabla \times \mathbf{E} = 0$ for both regions.

b. An illustration of the **E** and **B** fields can be obtained by running div_curl_example.m using matlab. Fig. 1 is the **B** field while Figure 3 is the **E** field. What are the properties of a field with non-zero curl?

Problem 3 - Stokes theorem

Calculate $\int (\nabla \times \mathbf{B}) \bullet \mathbf{ds}$ over the two surface areas enclosed by each path in Problem 1 (the shaded area). Compare your answer with the results from Problem 1a.

Problem 4 - Gradient

Compute the gradient of the following functions.

a. $f = 8\,a^2 \cos\phi + 2rz$ (cylindrical)

b. $f = a \cos 2\theta / r$ (spherical)

Use the worksheet associated with Problem 2.8.1 in "Visual Electromagnetics for Mathcad" to check your answer. (You may have to use a specific number instead of the variable *a*).

c. Calculate $\nabla \times \nabla f$ for each of the functions above.