Gauss' law

Reading assignment
Ulaby, 4-4
Connor and Salon, I-24 → I-29 and II-1 → II-10

Problem 1 - Coulomb and Gauss' law
Show that the electric field of a point charge satisfies Gauss' law by evaluating $\oint \mathbf{E} \bullet \mathbf{ds}$ over the surface of a sphere of radius *a*.

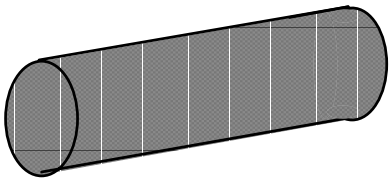
Problem 2 - Symmetry
Three charge distributions are pictured below. In 1) and 3), assume that the system is very long and ignore fringe effects. For each of the charge distributions, answer the following:

a. Determine the direction in which **E** points.

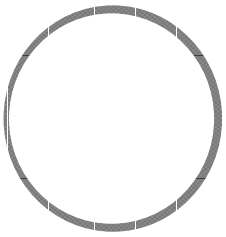
b. Determine surfaces over which $\int \mathbf{E} \bullet \mathbf{ds}$ is constant and non-zero.

c. Sketch a surface that can be used with Gauss' law to find **E**.

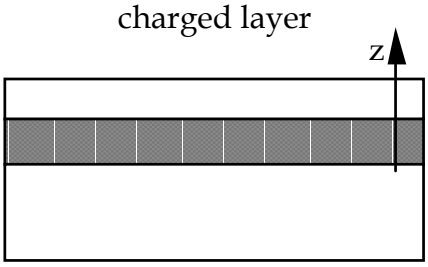
1) cylinder with uniform volume charge density



2) spherical shell of charge



3) semiconductor with charged layer



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Problem 3 - Use Gauss' law to evaluate E

A charge distribution with *cylindrical* symmetry is shown below. The inner cylinder has a uniform charge density ρ_v C/m^3. The outer shell has a surface charge density ρ_s C/m^2 such that the total charge on the outer shell is the negative of the total charge in the inner cylinder. Ignore end effects.

a. Find **E** for r < *a* .

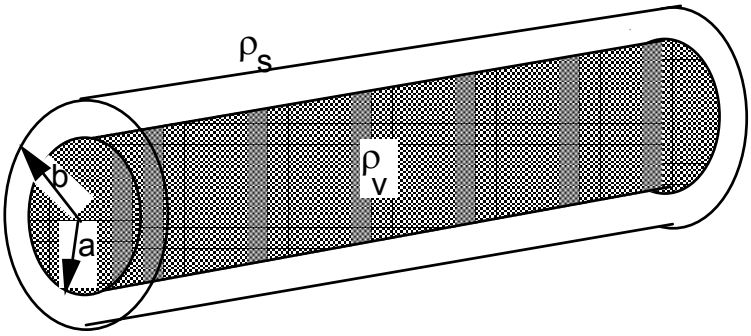
b. Find **E** for *a* < r < *b*

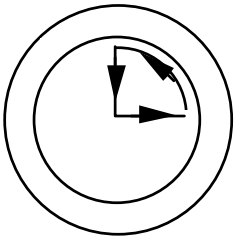
c. Find **E** for *b* < r.

d. Check your answer for **E** by evaluating $\nabla \bullet \mathbf{E}$ (the differential form of Gauss's Law) and $\nabla \times \mathbf{E}$ for all regions.

e. What is $\oint \mathbf{E} \bullet \mathbf{dl}$ around the closed contour shown on the right?

f. Express the unknown charge density ρ_s in terms of the geometry and the known uniform charge density ρ_v.





integration contour for part e.