Introduction to Transmission Lines

Reading assignment
Ulaby, 2-1, 2.2, 2.4

Experiment 1 - Compare lumped delay line and coaxial cable.
Obtain 1) a lumped model of a transmission line with 390 pF capacitors and 1 μH inductors, 2) a coaxial cable spool, and 3) a 50 Ω terminator. Set the function generator to produce a 600 kHz sine wave.
a. Put the 50 Ω terminator across the output of the coaxial cable and simultaneously measure input and output signals on the oscilloscope.
Measure the time delay between the signals. What else is different about the input and output signals?

b. Replace the coaxial cable with the lumped model transmission line.
1) Measure the time delay between the input and output signals.
Compare with your coaxial cable measurement.
2) Measure the time delay between the input and several other nodes.

Problem 1 - Coaxial cable L & C
The inductance per unit length of a coaxial cable is found to be $l = (\mu_0/2\pi)\{\ln(b/a) + 0.25\}$. For these calculations, drop the internal inductance term and use $l = (\mu_0/2\pi)\ln(b/a)$. The capacitance per unit length is calculated as $c = 2\pi\varepsilon / \ln(b/a)$. (The notation is the same as the book where script lower case l and c indicate per unit length variables).
a. Numerically evaluate l and c for an RG 58A/U coaxial cable which has a polyethylene dielectric ($\varepsilon_r = 2.3$) and copper conductors with an inner radius, $a \approx 0.4$ mm and an outer radius, $b \approx 1.4$ mm.
b. To model a 4 meter section of a coaxial cable line, what size L and C would you use? Compare with the values used in the experiment. What length of coaxial cable is equivalent to the lumped model in the experiment?

Problem 2 - Propagation velocity - revisit experiment 1
a. Let $V = V_0 \cos(\omega s)$ where $s = t \pm (z/u)$ and u is a propagation velocity. Substitute this expression into eq (24a) and solve for u in terms of l and c.
b. Substitute the expressions for l and c of a coaxial cable into the formula of the previous section and find u in terms of the material properties.
c. Numerically, what is the velocity of propagation in RG 58 A/U cables?
d. Use the time delay measured in the experiment to determine the length of your coaxial spool.
e. For the lumped model, calculate the time delay per section you expect. Compare your measured time delays with what you calculate.

Problem 3 - Characteristic impedance

a. Substitute $V = V_0\cos(\omega s)$ and $I = I_0\cos(\omega s)$ into equation (2.14 or 2.16) where $s = t \pm (z/u)$. Find R_C or $Z_0 = |V_0/I_0|$, the characteristic impedance of the transmission line. Write the answer in terms of l and c.

b. Numerically evaluate R_C or Z_0 for an RG 58 A/U cable.

Problem 4 (Optional) - Time varying fields in a coaxial cable

In electrostatics, $\mathbf{E} = \rho_{s0}\, a / (\varepsilon\, r)\, \mathbf{a}_r$ for a coaxial cable where ρ_s is the surface charge density on the inner conductor. In magnetostatics, $\mathbf{H} = I/(2\pi r)\, \mathbf{a}_\varphi$ for the same geometry. For a time varying problem, you can obtain the correct fields by replacing ρ_s with $\rho_{s0}\, f(s)$ and I with $I_0\, f(s)$ where f is any function and $s = t - (z/u)$ with the velocity $u = \pm(\mu_0\, \varepsilon)^{-0.5}$

a. Assume a time varying form of $f(s) = \cos(\omega s)$ with $s = t - z/u$ and show that Maxwell's equation for $\nabla \times \mathbf{H}$ is satisfied if you assume that $I_0 = 2\pi a u\, \rho_{s0}$.

b. (if time permits) Show that the other 3 Maxwell equations are also satisfied.

c. Derive the relation, $I_0 = 2\, \pi\, a\, \rho_{s0}\, u$. To do this simply determine the charge flow that results from ρ_s moving down the conductor at a velocity u.

d. Find the voltage difference $= -\int \mathbf{E} \bullet \mathbf{dl}$ between the inner and outer conductor.

e. Find R_C or $Z_0 = V/I$ using the result of part d. Evaluate numerically for an RG 58A/U cable.