

Reading assignment

Ulaby, 3-4

Connor and Salon, II-39 → II-44

Software

div_curl_example.m

Problem 1 - Line integrals & curl

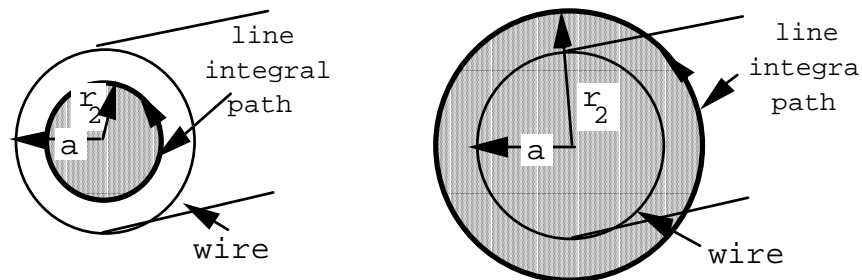
The magnetic field of a straight wire of radius a which has a constant current density J_0 , is given by:

$$\mathbf{B} = \mu_0 J_0 r / 2 \mathbf{a}_\phi \quad \text{inside the wire } (r < a)$$

$$\mathbf{B} = \mu_0 J_0 a^2 / (2 r) \mathbf{a}_\phi \quad \text{outside the wire } (r > a).$$

where μ_0 and J_0 are constants.

- a. Calculate $\oint \mathbf{B} \cdot d\mathbf{l}$ around the 2 paths shown in the figure below. (The drawing shows a view as if the wire had been cut).



- b. Calculate $\nabla \times \mathbf{B}$ for both regions.

Problem 2 - Properties of fields with curl

The electric field created by a cylinder of radius a with constant charge density ρ_0 is:

$$\mathbf{E} = \rho_0 r / (2 \epsilon_0) \mathbf{a}_r \quad \text{inside the cylinder } (r < a) \text{ and}$$

$$\mathbf{E} = \rho_0 a^2 / (2 \epsilon_0 r) \mathbf{a}_r \quad \text{outside the cylinder } (r > a).$$

where ρ_0 and ϵ_0 are constants.

- a. Verify that $\oint \mathbf{E} \cdot d\mathbf{l} = 0$ on the same path as above and that $\nabla \times \mathbf{E} = 0$.
- b. An illustration of the \mathbf{E} and \mathbf{B} fields can be obtained by running `div_curl_example.m` using matlab. Fig. 1 is the \mathbf{B} field while Figure 3 is the \mathbf{E} field. What are the properties of a field with non-zero curl?

Gradient, Line integrals, & Curl

Problem 3 - Stokes theorem

Calculate $\int (\nabla \times \mathbf{B}) \cdot d\mathbf{s}$ over the surface area enclosed by the 2 paths in Problem 1 (the shaded area). Compare your answer with the result from Problem 1a.

Problem 4 - Gradient

Compute the gradient of the following functions.

a. $f = 8a^2 \cos \phi + 2rz$ (cylindrical)

b. $f = a \cos 2\theta / r$ (spherical)

Use the worksheet associated with Problem 2.8.1 in "Visual Electromagnetics for Mathcad" to check your answer. (You may have to use a specific number instead of the variable a).

c. Calculate $\nabla \times \nabla f$ for each of the functions above.