

Surface integrals and divergence

Reading assignment

Ulaby, 3-5

Connor and Salon, II-26 → II-34

Software

div_curl_example.m

Problem 1 - Surface integralsCalculate $\int \mathbf{A} \cdot d\mathbf{s}$ for each of the following cases.

- $\mathbf{A} = 3 \mathbf{a}_r$, surface is $r = 3$, $0 \leq \varphi \leq \pi/3$, $-2 \leq z \leq 2$.
- $\mathbf{A} = 2r \mathbf{a}_r + 6r \mathbf{a}_\varphi$, surface is $0 \leq r \leq 5$, $\theta = \pi/3$, $0 \leq \varphi \leq 2\pi$.

Problem 2 - DivergenceCalculate $\nabla \cdot \mathbf{A}$ for each of the vectors below.

- $\mathbf{A} = x^2y \mathbf{a}_x + c^2x \mathbf{a}_z$
- $\mathbf{A} = c/r^2 \mathbf{a}_r + e^{-j\beta r} \sin\theta/r \mathbf{a}_\varphi$

c and β are constants. Use the worksheet associated with Problem 2.10.4 in "Visual Electromagnetics for Mathcad" to check your answer. (You may have to use specific numbers instead of the variables c and β .)

Problem 3 - Divergence theoremShow that the divergence theorem is valid by calculating $\int (\nabla \cdot \mathbf{A}) dv$ and

$\int \mathbf{A} \cdot d\mathbf{s}$ for the vector \mathbf{A} of Problem 2a. The volume integral should be for a cube with sides of length l as shown below.

