

Capacitance & Energy

Reading assignment

Ulaby, 4-10, 4-11

Connor and Salon, IV-6 → IV-30

Problem 1 - capacitance and energy

In Problem 2 from Lesson 2.4, we studied a coaxial cable with inner radius, a , and outer radius, b , length l . The outer conductor was grounded, and inner conductor had voltage $V = V_0$. The relation between surface charge density on the inner conductor and V_0 is

$\rho_{sa} = \epsilon V_0 / (a \ln(b/a))$. The electric field is given by $\mathbf{E} = V_0 / (r \ln(b/a)) \mathbf{a}_r$.

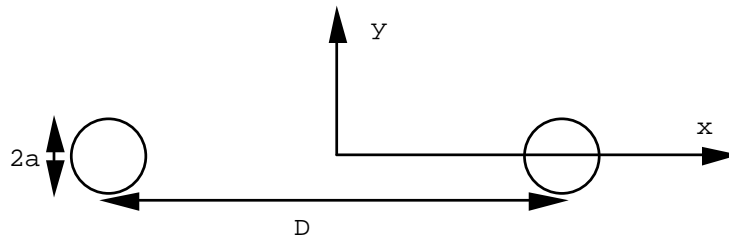
- Find the capacitance using $C = Q/V$.
- Find the stored energy in the system by integrating the energy density over the system volume.
- Find the capacitance using the stored energy from part b.

Problem 2 - analytical calculation of capacitance

The objective of this problem is to determine the capacitance between 2 conducting wires. We will assume there are equal magnitude, opposite sign, uniform surface charge densities on the two wires. This yields an equation for the electric field on the x -axis between the two wires (the wires are parallel to the z -axis):

$$\mathbf{E} = \frac{\rho_s a}{\epsilon} \left\{ \frac{1}{x + \frac{D}{2}} + \frac{1}{\frac{D}{2} - x} \right\} \mathbf{a}_x \quad |x| < (D/2 - a) \text{ and } y=0$$

- What is the voltage difference between the wires?
- What is the capacitance per unit length?
- In reality, is the surface charge density on the outside of the wires uniform? If not, how is the charge distributed?
- What is the energy density of the field along the x axis, as a function of position?
- What is the stored energy in the system?



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Problem 3 - capacitance experiment

- Build a small capacitor using two parallel wires. Estimate its value using the results from Problem 2. If a capacitance meter is available, compare the measured and calculated values.
- Include the estimated capacitor as part of the circuit below. Analytically evaluate V_0 at $f = 30$ and 300 kHz (either by hand or using Spice). A key comparison is the ratio of voltages at the two frequencies and at what frequency the signal saturates.
- Measure the node voltages at the same frequencies and quantitatively compare it with your answer to part b. (Again the ratio of signals at two frequencies should be calculated). There is another capacitance in the problem that affects the result. Can you identify it?
- Put your hand against the two wires. Then take your other hand and touch ground. What changes when you touch ground vs. when you aren't touching it. Why? How would you include this effect in your circuit model?

