Currents and Resistance

Reading assignment
Ulaby, 4-7, 7-5
Connor and Salon III-4 6 III-4, VIII-4,5

Problem 1 - Current density

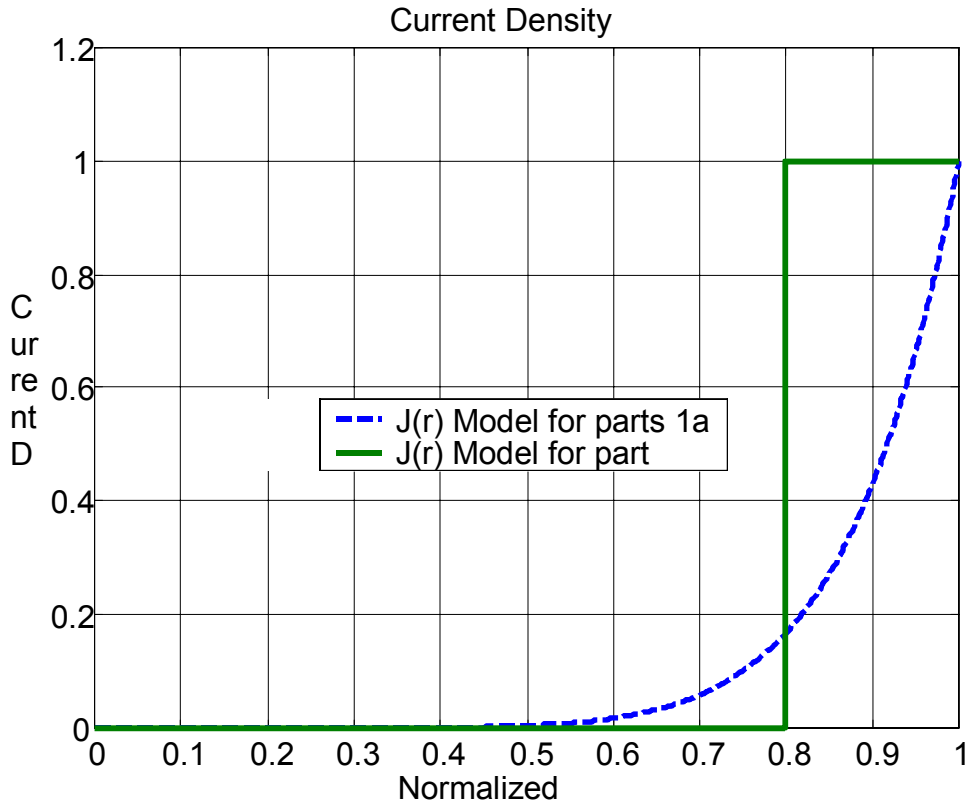
a. A copper wire carrying a current I has a current density given by

$\vec{J} = J_o\left(\frac{r}{a}\right)^8 \mathbf{a}_z$ for r < a and 0 for r > a.

Find J_0 in terms of a and I. Then get a numerical answer using I = 10 mA and a = 1 mm.

b. Notice how **J** is highly peaked toward the edge. As we will see later in the semester, the current density in a wire is peaked at the edge for high frequencies, although the actual functional form is different than used here. In the extreme high frequency limit, it can be useful to model the current as a surface current. What is the surface current density of a 1 mm radius wire carrying 10 mA?

c. If the charge density of the mobile electrons in the copper is 1.8 x 10^{10} C/m^3, what is the average drift velocity of the electrons under the conditions of part a?



Problem 2 - Resistance

For this problem, use basically the same geometry as in problem 1. Consider a 1 mm radius wire that is 10 meters long.

a. At DC frequencies, the current density is spread uniformly across the entire cross-section. What is the resistance of the wire?

b. A rough model for the wire in problem 1a is to assume that the current density is uniform in the region r > 0.88a and zero elsewhere, $\mathbf{J} = J_0 \mathbf{a}_z$ for r/a = 0.88-> 1 and 0 for r/a < 0.88 . What resistance is obtained with this model?

c. A more accurate value of the resistance is to calculate the voltage drop along the outside of the wire (at r = a) from $\mathbf{J} = \sigma \mathbf{E}$ and get the resistance from R = V/I. What does this model yield?

Experiment

Determine the resistance of a long spool of coaxial cable at 2 frequencies, 100 kHz and 10 MHz. Connect the experiment as shown below. Model the coax as a resistor in series with the terminator. Compare your experimental answer with what you would expect analytically. At 10 MHz, what fraction of the wire cross section is being effectively used?

The central conductor is copper with a radius of about 0.4 mm. Cable lengths are 80 ± 20 m.