Basic Math & Coordinate Systems

Reading assignment
Popović and Popović, Appendix 1
Connor and Salon, I-1 → I-14

Problem 1 - Dot and cross products
Given $\mathbf{A} = r^3 \mathbf{a}_r + 4\sin\theta \, \mathbf{a}_\varphi$ and $\mathbf{B} = 3 \mathbf{a}_r + 2 \mathbf{a}_\theta + 12 \sin\theta / r^3 \, \mathbf{a}_\varphi$
Find $\mathbf{A} \bullet \mathbf{B}$ and $\mathbf{A} \times \mathbf{B}$

Problem 2 - Area integrals
For each of the following surfaces, sketch the surface, and find its area.
a. $r = 3, \, 0 \le \varphi \le \pi/3, \, -2 \le z \le 2$.
b. $0 \le r \le 5, \, \theta = \pi/3, \, 0 \le \varphi \le 2\pi$.
c.   Identify **ds** for each of the surfaces.

Problem 3 - Volume integrals
a.   Sketch each of the following volumes and then calculate its value by integrating over the appropriate differential volume element.
        1) $2 \le x \le 5, \, 0 \le y \le 3, \, -2 \le z \le 3$.
        2) $1 \le r \le 3, \, 0 \le \varphi \le \pi/3, \, -2 \le z \le 2$.
b.   Integrate the function ($a \, e^{-r/a} / r$) over the volume of a sphere of radius a.

Problem 4 - Useful areas and volumes
a.   What is the surface area of a sphere of radius r?
b.   What is the surface area of the side of a cylinder with radius r and length l?
c.   What is the volume of a sphere of radius r?
d.   What is the volume of a cylinder of radius r and length l?