Surface integrals and divergence

Reading assignment
Popović and Popović, Appendix 1
Connor and Salon, II-26 → II-34

Software
div_curl_example.m

Problem 1 - Surface integrals

Calculate $\int \mathbf{A} \bullet \mathbf{ds}$ for each of the following cases.

a. $\mathbf{A} = 3\,\mathbf{a}_r$, surface is $r = 3$, $0 \le \varphi \le \pi/3$, $-2 \le z \le 2$.

b. $\mathbf{A} = 2r\,\mathbf{a}_r + 6r\,\mathbf{a}_\theta$, surface is $0 \le r \le 5$, $\theta = \pi/3$, $0 \le \varphi \le 2\pi$.

Problem 2 - Divergence

Calculate $\nabla \bullet \mathbf{A}$ for each of the vectors below.

a. $\mathbf{A} = x^2 y\,\mathbf{a}_x + c^2 x\,\mathbf{a}_z$

b. $\mathbf{A} = c/r^2\,\mathbf{a}_r + e^{-j\beta r}\sin\theta / r\,\mathbf{a}_\varphi$

c and β are constants. Use the worksheet associated with Problem 2.10.4 in "Visual Electromagnetics for Mathcad" to check your answer. (You may have to use specific numbers instead of the variables c and β.

Problem 3 - Divergence theorem

Show that the divergence theorem is valid by calculating $\int(\nabla \bullet \mathbf{A})\,dv$ and $\int \mathbf{A} \bullet \mathbf{ds}$ for the vector $\mathbf{A}$ of Problem 2a. The volume integral should be for a cube with sides of length l as shown below.