Electromagnetic Plane Waves in Lossless Media

Reading assignment

Popović and Popović, Chapters 19.21
Connor and Salon, Unit IX

Problem 1 - Intro to electromagnetic waves

The electric field of a plane wave is given by $\mathbf{E} = E_m \cos(\omega t - \beta z)\, \mathbf{a}_x$.

a. Write **E** in phasor form.

b. Under what conditions does **E** satisfy the wave equation (in phasor form), $\partial^2 E_x / \partial z^2 = -\mu\varepsilon\omega^2 E_x$?

c. What is the velocity of a plane wave in free space?

d. What are typical values of f, ω, T (=1/f), β, and λ for X-rays, visible light, microwaves, and FM radio in free space? (Check the walls of the studio).

e. Find **H** using the phasor form of the $\nabla \times \mathbf{E}$ equation. Assume the **E** and **H** phasors are only a function of z.

f. Evaluate the amplitude ratio, $\eta = |\mathbf{E}| / |\mathbf{H}|$. Express η in terms of material properties.

g. If **E** was in the $\mathbf{a}_y$ direction, what direction would **H** be in?

h. Run the Java applet linked through the course homepage.

i. How many independent parameters are there in the following set?
ω, β, μ, ε, η, λ, T

Problem 2 - Waves in lossless media

WRPI broadcasts at 91.5 MHz. The amplitude of **E** on campus is roughly 0.08 V/m. Assume a coordinate system in which the wave is polarized in the $\mathbf{a}_y$ direction and propagating in the $\mathbf{a}_z$ direction. Assume the phase = 0 at z = 0.

a. What are β, η and λ for this wave?

b. Write the electric and magnetic fields in phasor form.

c. Write the electric field in time domain form.

Problem 3 - Energy & Power - lossless media

a. What is the average energy density of the electric and magnetic fields for the WRPI signal on campus? (Use Prob. 2 results).

b. What is the time average Poynting vector of the wave, $\mathbf{S}_{av}$? Divide its magnitude by the speed of light and compare with your answer from part a.

c. The transmitter is about 10 km from campus. What transmitter power is required to radiate the same power density into a sphere of radius 10 km?