Review

Magnetic Fields

Example - Rectangular cross section torus

Side View

Top View

With this example we can review inductance B fields, Ampere's law, boundary conditions (often magnetic circuits).
Assume we want to know the field at some point.

We can use Ampere's Law since

\[ \mathbf{B} = \hat{\phi} B_\phi (r) \]
\[ \oint \mathbf{H} = \hat{\phi} H_\phi (r) \]

The field depends on many variables. This is critical for Ampere's Law. We can apply

\[ \oint \mathbf{H} \cdot d\mathbf{l} = I \text{ encircled on the path shown. (---)} \]

On this path, \( H \oint B \) are constants \( \Rightarrow \oint \mathbf{H} \cdot d\mathbf{l} = H_\phi (r) 2\pi r \)
for this path, we enclose all $N$ windings so that

$\text{Hence } = NI$

$N$ Turns wrapped around the core.

Thus $\int \overrightarrow{H} \cdot d\overrightarrow{A} = H_\phi(r) 2\pi r = NI$

or $\overrightarrow{H} = \hat{\phi} \frac{NI}{2\pi r}$

$\hat{\mu} \overrightarrow{B} = \hat{\phi} \frac{\mu NI}{2\pi r}$

for a core with $\mu = \mu_r \mu_0$. 
Simplifying $\mathbf{B}$ & $\mathbf{H}$: This is one of the most important steps in applying both Ampere's law & Gauss' law (for $\mathbf{E}$ fields).

Note - In this review we will emphasize the common features found in different parts of this course. Electromagnetism is not a collection of unrelated topics. What you learn in each part of the course will have some value in other parts of the course.

To apply Ampere's law or Gauss' law, the field must depend on only one variable.
To show that \( \vec{H} = \vec{H}(r) \)

go back to the long -

straight wire

\[ I \rightarrow \vec{H} \]

\[ \vec{H} \]

End View

For current into the page, the right hand rule gives vs the direction of \( \vec{H} \) shown. Since the long wire is specified only by \( r \), the field must also only depend on \( r \).
(The current exists in \( 0 \leq r \leq a \))
\[ \Rightarrow \quad \vec{H} = \hat{\phi} \cdot H_\phi (r) \]

The magnetic field of a long straight wire (see text) is
\[ \vec{H} = \hat{\phi} \cdot \frac{I}{2\pi r} \]

The evaluation of Ampere's law is the same
\[ \oint \vec{H} \cdot d\vec{l} = H_\phi (r) \cdot 2\pi r \]
\[ = I \text{ enc.} = I \]

Outside the wire \( \Rightarrow \quad \vec{H} = \hat{\phi} \cdot \frac{I}{2\pi r} \)

Note: Just, except for the \( N \), this is the same as for the torus. Why is this so?
This happens because: The square or rectangular cross-section turns to a coaxial cable are essentially the same configuration.

For a coax, the H field in the region between the inner and outer conductors is...
given by \( \vec{H} = \frac{\vec{j} I}{2 \pi r} \).

The outer conductor does not charge this. Rather, it just provides a boundary to define the edge of the field region. 

\( H \) exists here but not outside.

Let's assume that the coax is built with two very thin foil conductors at \( r = a \) and \( r = b \).

That is, the conductors are thin surfaces at \( r = a \) and \( r = b \).
We can model these as surface currents:

$$\vec{J}_{sa} = \frac{\vec{I}}{2\pi a}$$

$$\vec{J}_{sb} = \frac{\vec{I}}{2\pi b}$$

For the region $r > b$, we have no magnetic field $\vec{H} = 0$. Since $\vec{H}$ is in the $\phi$ direction, $\vec{H}$ is tangent to $r = b$. Thus the B.C. here is

$$H_{tan1} - H_{tan2} = J_s$$

0 outside
or \( H_{\text{tan}} = H_\phi = Js \)

Checking, we see that
\[
H_\phi = \frac{I}{2\pi b} = Js
\]

as it must. Thus the outer conductor not only helps create the field, it provides the B.C. so that there is no field for \( r > b \). Since we have assumed thin conductors at \( r = \frac{a}{2} \), \( \vec{H} = 0 \) for \( 0 < r < a \).
Going one step further, assume that we have thin waster like conductors connecting inner & outer conductors at each end.

Ends

\[ J_{se} \]

Side View

D axis of cable

End View

Note the current flowing radially at the ends.
The current flows radially in the ends, spreading out as \( r \) increases. In fact, \( \frac{\hat{r}}{J} = \frac{I}{2\pi r} \) at one end and \( \frac{\hat{\theta}}{J} = -\frac{I}{2\pi r} \) at the other end. Again this is exactly the B.C. necessary to support a field in the case if no field beyond the ends.

(Try the B.C. to see this.)

Note that a coax with end caps looks just like a
turns. The only difference is that there is only 1 turn. We could have made such a turn a toroidal form around which we wrap a layer of conducting foil. This explains why the fields for coax & a torus look the same except for the # of turns N.

\[ \vec{H} = \hat{\phi} \frac{NI}{2\pi r} \quad \vec{H} = \hat{\phi} \frac{I}{2\pi r} \]

\[ \vec{B} = \hat{\phi} \frac{M NI}{2\pi r} \quad \vec{B} = \hat{\phi} \frac{N_0 I}{2\pi r} \]
What can we do with \( \vec{B} \) once we know it?

 Flux through one turn.

\[
\psi_m = \int \vec{B} \cdot d\vec{s} = \frac{MNI}{2\pi} \int_0^h dz \int_0^{\frac{a+w}{r}} dr
\]

\[
= \frac{MNI}{2\pi} h \ln \frac{a+w}{a}
\]

Note: we could also have used \( b = a+w \)
The total flux linked by the toroidal configuration is

\[ \lambda = N \psi_m = \frac{\mu N^2 I}{2\pi} h \ln \frac{a + w}{a} \]

\[ = L I \]

\[ \Rightarrow L = \frac{\mu N^2}{2\pi} h \ln \frac{a + w}{a} \]

For a coaxial cable (at the left) the \( L \) per unit length is

\[ \frac{L}{2} = \frac{\mu_0 I}{2\pi} h \ln \frac{b}{a} \]

(You can work this up.)
Again note that this is the same as the terms without the $N^2$ term.

\[ L = \frac{\mu_0}{2\pi} \frac{2}{a} \Rightarrow \text{Force} = \frac{N^2}{2\pi \mu_0 a^2} \]

because $h$ is for force.

We can also find the $L$ from

\[ W_m = \frac{1}{2} N I^2 = \frac{1}{2} \int B \cdot H \, dv \]

Integrate over the volume of the field.

\[ \frac{1}{2} \int B \cdot H \, dv = \frac{1}{2} \int_0^{2\pi} \int_0^h \int_0^{2\pi} \frac{\mu N^2 I^2}{a (2\pi)^2} \, r \, dr \, d\phi \]

\[ = \frac{1}{2} 2\pi h \frac{\mu N^2 I^2}{1 (2\pi)^2} \ln \frac{b}{a} \]
or \( l = \frac{\mu \Phi_0}{2\pi} N^2 \ln \frac{b}{a} \)

as before.

For inductance \( \frac{1}{2} \) capacitance there are two general approaches. The most reliable is

\[
W_e = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \int \textbf{D} \cdot \textbf{E} \, dv
\]

\[
\frac{1}{2} W_m = \frac{1}{2} LI^2 = \frac{1}{2} \int \textbf{B} \cdot \textbf{H} \, dv
\]

called the energy method. We can also use \( Q = CV \) \( \frac{1}{2} \)

but flux linkage is sometimes hard
to apply inside current. We don’t know this problem for conductors E fields so external works well for capacitance.

There are 4 basic conductors that we can arrange with Ampere’s Law. Torus, Cone, Solenoid, Parallel Plate (Stroplin).

Just as the turns of core are. Similar the solenoid & pp. what are essentially the same.
A solenoid is formed by wrapping $N$ turns of wire around some cylinder. For this configuration,

$$L = \frac{\mu N^2 \pi a^2}{\ell}$$

When we have assumed that the core permeability is $\mu$. The general form is

$$L = \mu N^2 \frac{\text{Area}}{\text{Length}}$$
Compare with a stripline. From 
*ultralong* the inductance per unit 
length is 

\[ L = \frac{\mu_0}{W} \]

so that the inductance of a line 
of length \( L \) is 

\[ L = \frac{\mu_0 L}{W} \]
A side view

\[ \vec{J}_s \]

\[ \times \times \times \times \times \times \times \times \]

\[ \vec{J}_s \]

\[ \vec{J}_s \]

\[ \vec{J}_s \]

Where we have added end caps again to create a closed structure. The x's show the direction of the \( \vec{H} \) field. The area of the field is \( \Delta l \times \frac{1}{2} \) the depth is \( w \). Thus for the entire line:

\[ L = M_0 \frac{\Delta l}{w} = M_0 \frac{\text{Area}}{\text{Length}} \]

on the same as the solenoid.
One can also look at the B.C. in the same manner as we did with the Torus & Coax.

Back to Ampere's Law

If we calculate \( \vec{B}, \vec{H} \) inside a current, then we can check the answer by applying

\[
D \times \vec{H} = \frac{\partial \vec{E}}{\partial t} \quad \text{(Static)}
\]

to see if we get back the original \( \vec{J} \) (Source)
We can do the same thing for $\mathbf{D}$, $\mathbf{E}$ by taking

$$\mathbf{D} \cdot \mathbf{A} = \Phi$$

**Basic Capacitors**

- Spherical Cap
- Coax
- Parallel Plate

From charge and find $\mathbf{D}$ using Gauss' Law

$$\oint \mathbf{D} \cdot d\mathbf{A} = \Phi$$
From \( D \rightarrow E = \frac{D}{\varepsilon} \)

From \( E \)

\[ V(b) - V(a) = -\int_a^b E \cdot d\ell \]

\( \uparrow \)

usually choose to be \( V(a) = 0 \)

\[ \begin{align*}
Q & \rightarrow D \rightarrow E \rightarrow V \\
\_\_\_\_ C & \leftarrow \_\_\_\_ \text{ Obtain Capacitance}
\end{align*} \]

Can also use \( W_e = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C} \)

\[ = \frac{1}{2} \int \bar{E} \cdot \bar{D} \, dv \]
We use \( J \) instead of \( E \).

In application of Gauss' law becomes, when all aspects of a configuration are given by a single coordinate (e.g. \( r \) for cylindrical or \( \rho \) for spherical or \( x \) for parallel plates), then \( J \) does not see the dielectric boundaries.

To go beyond the very simple geometries where we can address with Gauss' law, we need a numerical method.
The speed sheet method gives us \( V \) everywhere from which we can estimate \( \vec{E} \) and \( \vec{B} \) which we can use to find \( \rho_s \) through the B.C. at conductors. From there find the charge on each conductor.

\[
V \rightarrow \vec{E} \rightarrow \vec{D} \rightarrow \rho_s \rightarrow \mathcal{Q}
\]

\[\mathcal{C}\]

Find Capacitors this way
Note - You should know the basics of the grand staff method.

Transversal Lines

We have already discussed the similarities between transversal lines & uniform plane waves normally incident on prismatic boundaries. Be sure to review this material.
Basic Trans Line Config

![Diagram]

Source and Load

For the line, \( L / \rho C \) can be given, then \( Z_0 = \sqrt{\frac{L}{C}} \) \( u = \frac{1}{\sqrt{LC}} \) or \( Z_0, u \) can be given.

The source can be sinusoidal or pulsed. For both cases the key is \( Z_0 \).
For parallel, \( Z_{in} = Z_0 \)

For series, \( Z_{in} = \frac{Z_c + jZ_0 \tan \beta d}{Z_0 + jZ_c \tan \beta d} \)

The input voltages are given by voltage divided action at the source. For a lossless line, power will be delivered to the load.

Be sure you can take a simple trace line and analyze it for both generalized and practical input.
The fudgaz impacts would either be shown from the transit time \( T \) or very much longer (as in switching on a D.C. source).

Note - these comments were motivated by looking at HW8 for Spring 2005 \( \frac{1}{8} \) 8 extra credit questions for Spring 2005