Transmission Lines
Steady State

Phasor Notation

$$V(z,t) = \mathrm{Re}(\hat{V}(z)e^{jwt})$$
$$I(z,t) = \mathrm{Re}(\hat{I}(z)e^{jwt})$$

Voltage Wave

$$\hat{V}(z) = \hat{V}_m^{\,+}e^{-gz} + \hat{V}_m^{\,-}e^{+gz} = \hat{V}_m^{\,+}e^{-gz}(1+\hat{\Gamma}(z)) \text{ for lossy lines}$$
$$\hat{V}(z) = \hat{V}_m^{\,+}e^{-jbz} + \hat{V}_m^{\,-}e^{+jbz} = \hat{V}_m^{\,+}e^{-jbz}(1+\hat{\Gamma}(z)) \text{ for lossless lines}$$

Current Wave

$$\hat{I}(z) = \frac{\hat{V}_m^{\,+}}{Z_o}e^{-gz} - \frac{\hat{V}_m^{\,-}}{Z_o}\hat{V}_m^{\,-}e^{+gz} = \frac{\hat{V}_m^{\,+}}{Z_o}e^{-gz}(1-\hat{\Gamma}(z)) \text{ for lossy lines}$$
$$\hat{I}(z) = \frac{\hat{V}_m^{\,+}}{Z_o}e^{-jbz} - \frac{\hat{V}_m^{\,-}}{Z_o}\hat{V}_m^{\,-}e^{+jbz} = \frac{\hat{V}_m^{\,+}}{Z_o}e^{-jbz}(1-\hat{\Gamma}(z)) \text{ for lossless lines}$$

Propagation Constant

$$\hat{g} = a + jb = \sqrt{(R+jwL)(G+jwC)}$$

Characteristic Impedance

$$Z_o = \sqrt{\frac{R+jwL}{G+jwC}}$$

Phase Velocity

$$v_{ph} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{me}} \text{ for lossless or low loss lines}$$

Total Wave Impedance

$$\hat{Z}(z) = \frac{\hat{V}(z)}{\hat{I}(z)} = Z_o\frac{1+\hat{\Gamma}(z)}{1-\hat{\Gamma}(z)}$$

Reflection Coefficient

$$\hat{\Gamma}(z) = \frac{\hat{V}^-{}_m}{\hat{V}^+{}_m}e^{2gz} = \hat{\Gamma}(z')e^{2g(z-z')} \text{ for lossy lines}$$
$$\hat{\Gamma}(z) = \frac{\hat{V}^-{}_m}{\hat{V}^+{}_m}e^{j2bz} = \hat{\Gamma}(z')e^{j2b(z-z')} \text{ for lossless lines}$$

$$\text{Reflection from load } \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\text{Reflection from generator } \Gamma_g = \frac{Z_g - Z_o}{Z_g + Z_o}$$

Input Impedance of a Transmission Line of Length d with load Z_L

$$Z_{in}(z = 0) = Z_o \frac{Z_L + Z_o \tanh \gamma d}{Z_o + Z_L \tanh \gamma d} \text{ for a lossy line}$$

$$Z_{in}(z = 0) = Z_o \frac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} \text{ for a lossless line}$$

Z_o is real when the line is lossless

Average Power

$$P_{ave}(z) = \frac{1}{2}\mathrm{Re}(\hat{V}(z)\hat{I}^*(z))$$

Standing Wave Ratio

$$SWR = \frac{|V_{\max}|}{|V_{\min}|} = \frac{|I_{\max}|}{|I_{\min}|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Transients

Lossless Wave Equation

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2}$$

$$\frac{\partial^2 I}{\partial z^2} = LC\frac{\partial^2 I}{\partial t^2}$$

Voltage and Current Waves

$$V(z,t) = V^+\left(t - \frac{z}{v_{ph}}\right) + V^-\left(t + \frac{z}{v_{ph}}\right) \equiv V^+ + V^-$$

$$I(z,t) = \frac{V^+}{Z_o} - \frac{V^-}{Z_o}$$

Z_o is also called the surge impedance