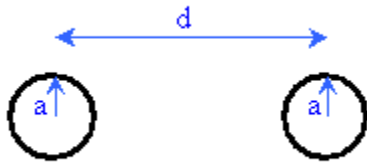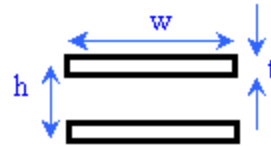*Supplemental Notes*
Two-Wire and Parallel Plate Transmission Lines
Version 1.0

Two-Wire Trans. Line   **Parallel Plate Trans. Line**

These two very common transmission lines are addressed several places in the textbook. However, the inductance analysis in each case is not necessarily very complete. Good pictures of the usual geometries for these and other standard lines are found on pages 424, 425 and 435. The parallel plate transmission line is more often found in the form shown on page 425. However, its analysis is essentially the same as that found on page 435 and shown in the figure above. Since this line looks like two long strips of copper, it is called a stripline. The two-wire line is most often found twisted and is called a twisted-pair line.

**Stripline or Parallel Plate Transmission Line Inductance Analysis**

The simplest model of this line is two parallel surface currents $\vec{J}_s = \pm \dfrac{I}{w}\hat{a}_z$, where the positive current flows axially in the top plate and the negative current flows in the bottom plate. (The geometry of the plates is shown in figure 7.7. The plate separation in figure 7.7 is s while the plate separation above is h. We will use h in this analysis.)

The field inside the parallel plate structure is like that of a solenoid in that it is constant throughout the region between the plates. Remember the approximation used for the parallel plate capacitor, in which the width w and depth (really long here) are assumed to be many, many times as large as the plate separation h. Then we can neglect fringing and we get the very simple field shown in the figure below. Since the current goes into the page in the top plate and out of the page in the bottom plate, the magnetic field (either $\vec{B}$ or $\vec{H}$) will be constant and point to the left (negative y direction). The magnetic field intensity is $\vec{H} = -\dfrac{I}{w}\hat{a}_y$, which one can find either from direct application of Ampere's Law or boundary conditions. To find the inductance, we must first determine the flux linked by this circuit. Rather than finding all of the flux, we will just find the amount of flux linked in a unit length of the line and then find the inductance per unit length. Since the flux points in the –y direction, we will choose our surface element to be in the same direction $d\vec{S} = -\hat{a}_y\,dxdz$. Then the flux will be given by

$$\psi_m = \int \vec{B}\cdot d\vec{S} = \int_0^1 dz \int_0^h dx\,\mu_o \frac{I}{w} = \mu_o I \frac{h}{w}.$$ Since there is only one turn in this case, $\Lambda = \psi_m$.

Thus, $L = m_o \dfrac{h}{w}$, which is in agreement with the expression given in equation 7-21a on page 436. Remember that this is the inductance per unit length.

B or H

x

y

z into the page
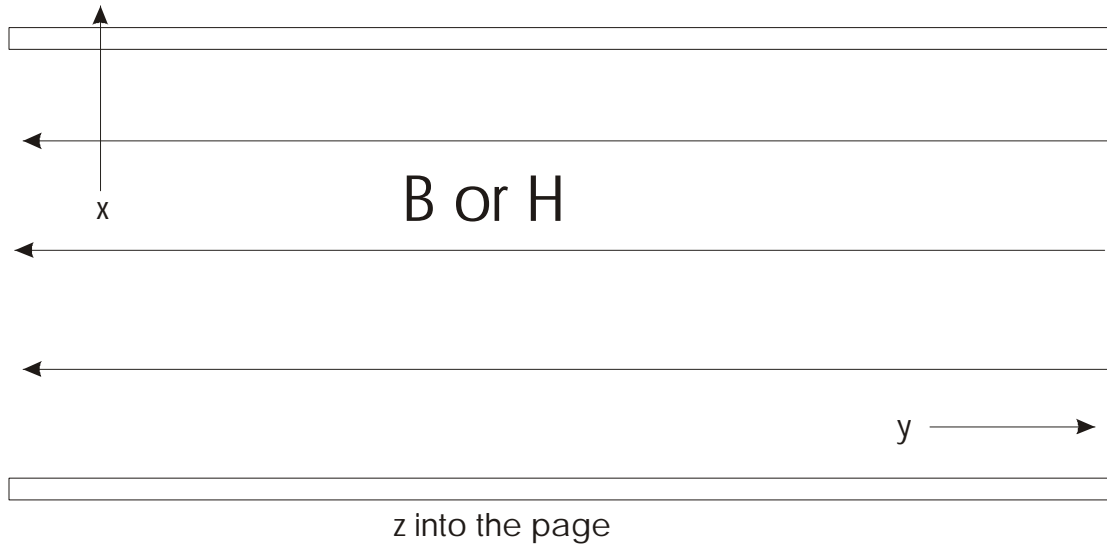
We will see when we get to transmission lines that the inductance and capacitance per unit length are related by the expression $u = \dfrac{1}{\sqrt{LC}}$, where u is the speed of propagation on the line (the speed of light) and L and C are the inductance and capacitance per unit length. Thus, it is usually possible to find one of these terms if we know the other and the speed of propagation.
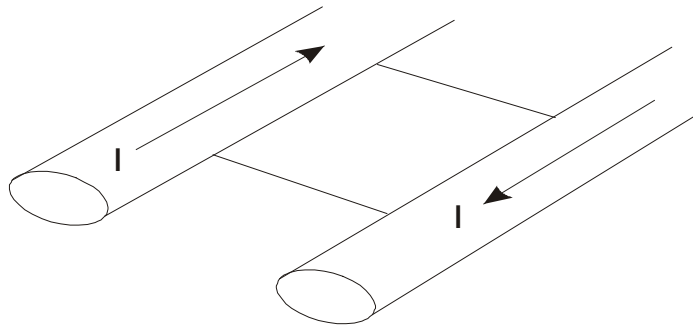
Two-Wire Transmission Line

The two-wire transmission line is just two long-straight wires running parallel to one another. Thus, each produces a magnetic field of the form $\vec{B} = \dfrac{m_o I}{2pr} \hat{a}_f$. The flux linked by the circuit is the flux found in the region between the two wires. (We are only concerned with the external inductance here.) Thus, the area we must integrate over to get the flux is of length l and looks like the rectangle bounded by the edge of the two wires shown below. The flux is given by the integral

$$y_m = \int \vec{B} \cdot d\vec{S} = 2 \int_a^{d-a} \frac{m_o I}{2pr} dr \int_o^l dz = 2 \frac{m_o I}{2p} \ln \frac{d-a}{a} = LI$$, where the factor of 2 is included to

show that each wire contributes the same flux. Thus, the inductance is $L = \dfrac{m_o}{p} \ln \dfrac{d-a}{a}$.

This expression is only approximately correct, since it assumes that the current is uniform in the wires. Two wires near one another will result is a small variation of the current from uniform. The error due to this approximation becomes quite small when the wires are far apart. Then the inductance given in chapter 7, equation 182b agrees exactly with the value we have just obtained.

References:

http://emcsun.ece.umr.edu/new-induct/wire2.html Two-wire inductance calculator java applet from the University of Missouri-Rolla.

http://emcsun.ece.umr.edu/new-induct/trace-v.html Inductance calculator for two vertically spaced printed circuit board traces from the University of Missouri-Rolla.

Some Stripline Configurations Found in Practice: