Fields and Waves I

Name _______________ ECSE-2100 Spring 2000 Section ____________

Preparation Assignments

Due at the start of class.

Reading Assignments

Please see the handouts for each lesson for the reading assignments.

10 January First day of class.

Due 12 January (2 points) Lesson 1.2 & Lesson 1.3 Problem 4

1. **A** = 3x $\mathbf{a_x}$ + 2z $\mathbf{a_y}$ + 4z^2 $\mathbf{a_z}$ and **B** = 3y $\mathbf{a_x}$ – 5 $\mathbf{a_y}$

   What is **A· B?**

   What is the unit vector in the direction of **B**?

2. What is the differential volume element in cylindrical coordinates?

4. What is a typical delay time for the reels of coaxial cable used in the studio?

Due 13,14 January (2 points) Lesson 1.3 & Lesson 1.4 Problem 1

A = 5x^2 $\mathbf{a_x}$ + (3y+2x) $\mathbf{a_y}$ +z^2 $\mathbf{a_z}$

1. What is the line integral $\int \vec{A}\cdot d\vec{l}$ along the path from the point (1,2,3) to the point (1,4,3)?
2. What is $\nabla \times \vec{A}$?
3. What is the complete area of all surfaces a cylinder of radius r and length L?
4. What is the volume of a cylinder of radius r and length L?
5. What is the area of a sphere of radius r?

17 January Holiday

Due 19 January (2 points) Lesson 1.4 & Lesson 2.1

Using the same expression for **A** as in the previous assignment,

1. What is $\nabla \cdot \vec{A}$?
2. If $\int \nabla \cdot \vec{D} dv$ integrated over some volume is equal to 5 coulombs, what is the value of $\oint \vec{D}\cdot d\vec{s}$ integrated over the surface of the volume?
3. What is $\int \nabla \times \vec{A}\cdot d\vec{S} = \oint \vec{A}\cdot d\vec{l}$ equal to when $\vec{A} = 5\hat{a}_x$?
4. What is $\nabla \times \nabla f$ for $f = ax + by + cz$?

Class time 20,21 January

Open shop to work on Homework 1. Due at 5 pm on 21 January.

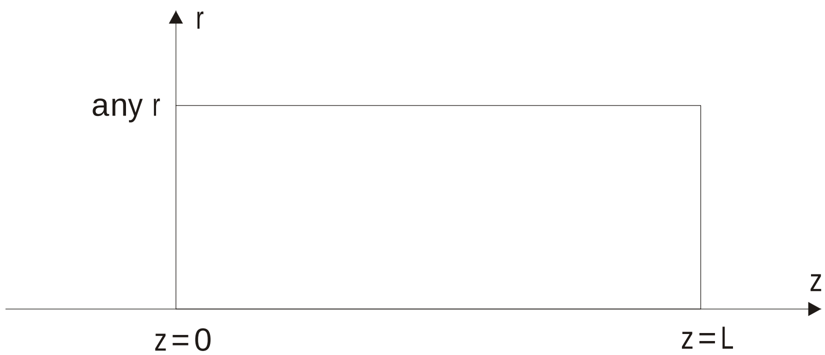
Homework #1

Problem 1 – (10 Points)

A spinning cylindrical object carries a current density in the region r = a given by $\vec{J}(r) = \hat{a}_f J_o \frac{r}{a}$. (This expression holds for all f and all z.) The current density is zero everywhere else. That is, it is zero for r > a.

a. Plot the magnitude of the current density as a function of r.

We want to determine the current passing through an area of length L between the z-axis and some arbitrary radius r. Until you are told otherwise, assume that r < a.



b) First, what is the vector surface element for this surface? Circle the correct answer.

$$d\vec{S}_z = \hat{a}_z r df dr, d\vec{S}_r = \hat{a}_r r df dz, d\vec{S}_f = \hat{a}_f dr dz$$

c) Simplify the following integral (for the total current passing through the surface S we just defined) and then evaluate it. $I(r) = \int_S \vec{J} \cdot d\vec{S}$ Note that the integral is given in terms of I(r), since it is a function of r. Begin by evaluating the dot product inside the integral using the surface element you selected above.

d) The answer you just obtained should hold only for r < a. Now assume that r >a. Evaluate the same integral. Now the answer will be equal to the total current I_{Total} in the region between z = 0 and z = L, since there is no current outside r = a.

e) When we get to magnetic fields, we will find that the magnetic field for r < a of this current distribution is given by $H_z(r) L = I_{Total} - I(r)$. Evaluate this expression to find $H_z(r)$ for r < a, using your answers to (c) and (d).

f) We will also find that $\nabla \times \vec{H} = \vec{J}$. Evaluate the curl of your answer to (e) and show that you get back the original expression for current density.

Problem 2 – (10 points)

We are given a spherical charge distribution $r(r) = r_o\left(1 - \frac{r}{a}\right)$ for r < a and $r = 0$ elsewhere.

a) The integral $Q_{Enclosed} = \int_V r\,dv$ is the charge contained in a volume V defined by a closed surface S. Thus, it is the charge enclosed by the surface S. Let V be a spherical volume of radius r, where r < a. Determine the charge contained in this volume $Q(r)$. Begin by writing out this integral fully with its limits and then evaluate it.

b) Now evaluate the integral for a sphere of radius r > a. Since r > a, the integral will now equal the total charge Q_{Total} that exists anywhere.

c) When we consider Gauss' Law for electric fields, we will see that $D_r(r)4pr^2 = Q(r)$ will give us the electric flux density in the region r < a. We will also see that $\nabla \cdot \vec{D} = r$. Evaluate $D_r(r)$ using the formula above and your answer for Q(r). Then show that the divergence formula is satisfied and, thus, that your answer for Q(r) is correct.

d) Your answer for $D_r(r)$ can be used to determine the electric field $E_r(r)$, since they are usually related by a constant. In empty space (also called free space), $\vec{D} = e_o\vec{E}$. The electric field can also be found from the electric scalar potential V using $\vec{E} = -\nabla V$. For the charge distribution used in this problem, $V(r) = \frac{Q_{Total}}{4pe_o a} - \frac{r_o}{e_o}\left[\frac{r^2}{6} - \frac{r^3}{12a}\right] + \frac{r_o}{e_o}\frac{a^2}{12}$.
Using your answers to the previous parts of this problem, show that this is the correct expression for the electric scalar potential (also called the voltage).