Preparation Assignments

Due at the start of class

Reading Assignments

Please see the handouts for each lesson for the reading assignments.

8 January First day of class

10 January (5 points) Lesson 1.2 and 1.3 Problem 4

1. $\vec{A} = 3x\hat{a}_x + (2z + y)\hat{a}_y + 4z^2 a_z$ and $\vec{B} = 3y\hat{a}_x - 5z\hat{a}_y$

   What is $\vec{A} \cdot \vec{B}$?

   What is the unit vector in the direction of $\vec{B}$?
2. What is the differential volume element in spherical coordinate?
3. How many turns of wire were used in the $1mH$ inductor used in Lesson 1.1?
4. If 9 reels of coaxial cable (like the ones used in the studio) were connected together in series, what would be the approximate time for a pulse to propagate from one end to the other?
5. Is the core of the $1mH$ inductor magnetic or not? How can you tell?

Due 11 January (5 points) Lesson 1.3 and Lesson 1.4 Problem 1

$\vec{A} = 5x\hat{a}_x + (3y^2 + 2x)\hat{a}_y + z^2 a_z$

1. What is the line integral $\int \vec{A} \cdot d\vec{l}$ along the path from the point (1,2,3) to (1,2,4)?
2. What is $\nabla \times \vec{A}$?
3. What is the complete area of all surfaces a hemisphere of radius r?
4. What is the volume of a hemisphere of radius r?
5. What is the complete area of all surfaces of a cube of length=height=depth=a?

15 January Holiday

Due 17 January (5 points) Lesson 1.4 and 2.1

Using the same expression for $\vec{A}$ as in the previous assignment,

1. What is $\nabla \cdot \vec{A}$?
2. If $\int \nabla \cdot \vec{D} dv$ integrated over some volume is equal to 33 coulombs, what is the value of $\oint \vec{D} \cdot d\vec{s}$ integrated over the surface of the volume?
3. What is $\int \nabla \times \vec{A} \cdot d\vec{S} = \oint \vec{A} \cdot d\vec{l}$ equal to when $\vec{A} = 7x\hat{a}_x$?
4. What is $\nabla \times \nabla f$ for $f = ax + by^2 + cz$?
5. What is $\nabla \cdot \nabla \times \vec{A}$ equal to when $\vec{A} = 7x\hat{a}_x$?

Class time 18 January

Open shop to work on Homework 1. Due at 6 pm on 18 January.

Homework #1

Problem 1 – (10 Points)

To investigate just what the curl operator tells us, we will consider a magnetic field given by the expression $\vec{H} = H_1\hat{a}_f$. That is, it has a constant magnitude and is in the f direction. This expression is assumed to hold only for radii smaller than a, that is, $r \le a$.

a) Even though this vector field has a constant magnitude, it still has a nonzero curl. Find the curl of *H*, $\nabla \times \vec{H}$ for $r \le a$.

b) According to Maxwell and Ampere, the curl of the magnetic field tells us what current produced the field, since $\nabla \times \vec{H} = \vec{J}$ where the vector $\vec{J}$ is the current density. Write the vector expression for the current density in this region.

c) Using your result for the current density, find the total current passing through the circular region *z=0*, $0 \le r \le a$, $0 \le f \le 2p$. To find the current passing through a surface *S*, you must evaluate the integral $I_S = \int_S \vec{J} \cdot d\vec{S}$. First, what is the vector surface element for this surface?
Circle the correct answer.

$$d\vec{S}_z = \hat{a}_z r df dr, d\vec{S}_r = \hat{a}_r r df dz, d\vec{S}_f = \hat{a}_f dr dz$$

c) Simplify the integral (for the total current passing through the surface S we just defined) and then evaluate it. Begin by evaluating the dot product inside the integral using the surface element you selected above. For this question, the limits on your integrals should be constants and your result should also be a constant.

d) Now, using your expressions for $\vec{J}$ and $\vec{H}$, show that Stoke's Theorem is satisfied. That is, show that $\int \nabla \times \vec{H} \cdot d\vec{S} = \oint \vec{H} \cdot d\vec{l}$ where the line is a circle of radius *r* in the plane *z=0* and the surface is the circular disk defined by the circle.

Problem 2 – (10 points)

We are given that the electric field, everywhere is space, is given by
$\vec{E}(r) = E_1 \dfrac{r^2}{a^2}\hat{a}_r$ for $r \le a$ and $\vec{E}(r) = E_2 \dfrac{a^2}{r^2}\hat{a}_r$ for $r > a$. The coordinate system is spherical.

a) According to Maxwell and Gauss, the divergence of the electric field tells us what charge distribution produced the field. Thus find the charge distribution by evaluating $r = \nabla \cdot e_o \vec{E}$ Note that you must apply this expression in both regions and that e_o is fundamental constant called the permittivity of free space.

b) The integral $Q_{Enclosed} = \int_V r\,dv$ is the charge contained in a volume V defined by a closed surface S. Thus, it is the charge enclosed by the surface S. Let V be a spherical volume of radius r, where $r < a$. Determine the charge contained in this volume $Q(r)$. Begin by writing out this integral fully with its limits and then evaluate it. Note that for this question, r can take on any value from *0* to a.

c) Now evaluate the integral for a sphere of radius $r > a$.

d) When we consider Gauss' Law for electric fields, we will see that $D_r(r)4pr^2 = Q(r)$ will give us the electric flux density $D_r(r)$. Evaluate $D_r(r)$ using this formula.

d) Your answer for $D_r(r)$ can be used to determine the electric field $E_r(r)$, since they are usually related by a constant. In empty space (also called free space), $\vec{D} = e_o\vec{E}$. Show that your answer for $D_r(r)$ is consistent with the given electric field.