Fields and Waves I

Name _______________ ECSE-2100 Spring 2000 Section ____________

Preparation Assignments for Homework #2

Due at the start of class.

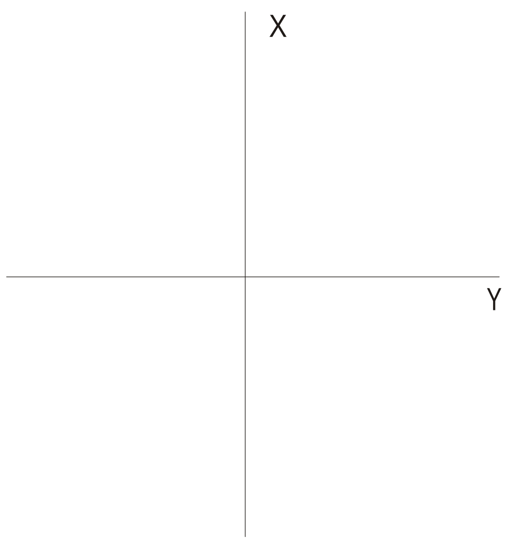
Reading Assignments

Please see the handouts for each lesson for the reading assignments.

Due 24 January (4 points) Lesson 2.2

Note: This assignment is not going to be collected, since it was not handed out in time. However, we will go over it in class. All students who attend class on the 24th will receive the 4 points.

1. Sketch a few electric field lines for a single positive point charge +Q and for an electric dipole field consisting of a positive point charge +Q and a negative point charge –Q separated by a distance d.
2. Show that the electric field of a point charge +Q located at the origin has no divergence, except possibly at the location of the charge itself.
3. One of the fundamental source distributions we will be working with in this course is the infinite sheet of charge. Assume that there is such a sheet located in the plane x = 0. Assume also that the surface charge density $\rho_s = \rho_{so}$ is a constant everywhere in the plane. Shown below is a two dimensional plot of the surface charge and its nearby region. Show where the sheet charge is located and then draw a few representative E field lines on this plot. Also write the vector expression for E in each region (x > 0 and x < 0).



4. State Gauss' Law for electric fields in your own words.

Due 26 January (4 points) Lesson 2.3

1. A point charge +Q is surrounded by a surface charge located at r = a. What must the surface charge density be to have no net charge in this configuration?

2. In what device that you use on a daily basis will you find something you could model as a uniform cylinder of charge?

3. Assume that the studio classroom is a parallel plate capacitor with one plate on the ceiling and one on the floor. If the electric field in the room is vertical and approximately equal to 100 volts per meter, what will the voltage be on the floor and the ceiling?

4. A point charge has a charge Q = 1 coulomb, how far away from it must you be to experience a voltage of 1 volt?

Class time 27 and 28 January
Open shop to work on Homework 2. Due at 5 pm on 28 January

Due 31 January (4 points) Lesson 2.4

1. For the electric dipole considered in the prep assignment for 24 January, what is the potential at the point exactly half way between the two charges?
2. If, instead of an electric dipole configuration, we had two identical positive charges +Q separated by a distance d, what is the potential at the point exactly half way between the two charges.
3. Sketch a few equipotentials for the electric dipole and for the configuration with the two identical positive charges.
4. Find the dielectric strength and the dielectric constant of polystyrene.

Homework #2

Problem 1 – (10 points) Gauss' Law

A charge distribution consists of a uniform surface charge $-\rho_{so}$ at the radius r = b and a volume charge density $r_v = \frac{Q}{pa^3} - \frac{Qr}{pa^4}$ for r < a and $r_v = 0$ elsewhere (for r > a).

where Q is an amount of charge in coulombs, r is the distance from the origin. Assume that the total volume charge equals the negative of the total surface charge and, thus, that the total charge in the problem is equal to zero. *Hint: Note that this charge distribution is quite similar, in part, to the distribution analyzed in the previous homework assignment.*

a) From the symmetry of the problem, identify the appropriate coordinate system, the direction of the electric field and the coordinates the field depends upon. Remember that, for the last question, you need to consider which coordinates the electric field does <u>not</u> depend on and that, for the problems we normally consider, the fields will vary with only one coordinate.

b) What is the density of the surface charge ρ_{so} such that the total of the positive and negative charges is zero?

c) Find the electric field at all locations (r >b, b > r > a and a > r) using Gauss' law. Be sure to identify or show the integrating surface that you use.

d) Using your answer for the electric field, find the voltage everywhere. (set $V(\infty) = 0$)

e) Check your answer using Poisson's equation, $\nabla^2 V = -\rho_v/\varepsilon$

f) Draw the geometry of this problem. On your drawing, include a small number of electric field lines and equipotentials (enough to be representative).

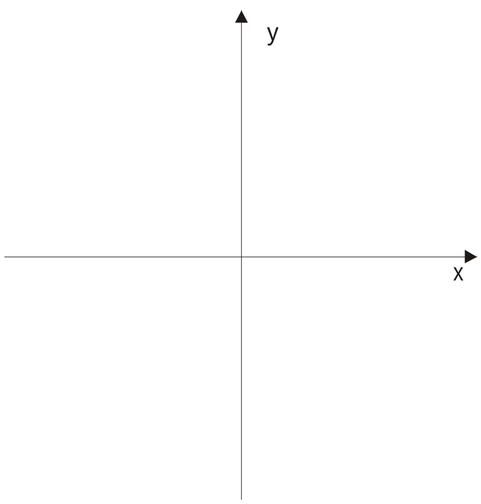
g) Plot the magnitude of the electric field and the voltage as a function of radius. (You should have a figure of the form shown in figure (b) for example 3.12.) *Hint: It might be best to do this using Matlab, Maple orExcel.*

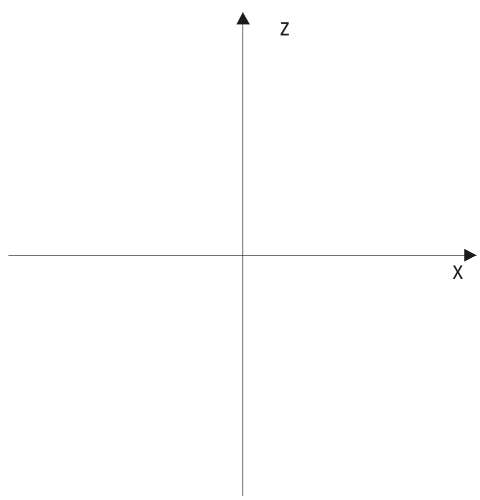
Problem 2 – (10 points)

If we knew exactly where all the charges are in some electrostatic configuration, it is relatively straight forward, if possibly tedious, to determine the resulting field. All we need do is to find E or V from Coulomb's law for each charge and then add up all the contributions. Computers allow us to actually approach problem solving in this manner, since they can free us from much of the work involved in this very general approach to evaluating fields. Still, it is difficult to know everything about the charges before we know everything about the fields. Many times, however, it is possible to guess a good deal about the charges before we try to figure out the fields. We will look at some simple charge distributions in both cylindrical and spherical coordinates and see what Coulomb's law can tell us about them. *On all plots below, indicate the locations of the charges.*

a) Before we proceed with this problem, write down the expressions for the electric field E and the electric potential V for a point charge +Q located at the origin. Then write the expressions for E and V for a constant line charge ρ_{Lo} located on the z-axis.

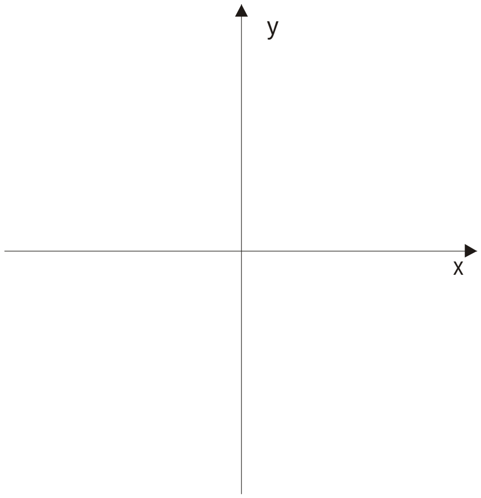
b) Now assume that we have two positive point charges +Q located at (x,y,z) = (±d/2,0,0). What direction does the field produced by both charges point on the x-axis, y-axis and z-axis? Show the directions below. Also show any locations where the electric field or the potential equal zero and evaluate V at the origin. Sketch 3 equipotentials. *Note: If the figure at the right will look identical to the figure at the left, it is not necessary to show everything twice. Also, be sure you show the charge locations.*

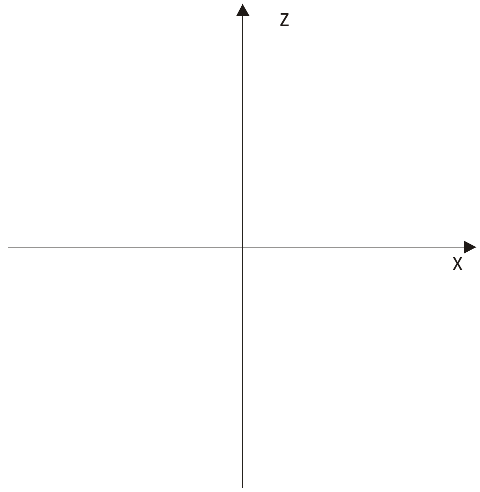




c) Assume that there are two positive line charges located at x = ±d/2, y=0, for all z. *Note: It is no longer necessary to do this part.*

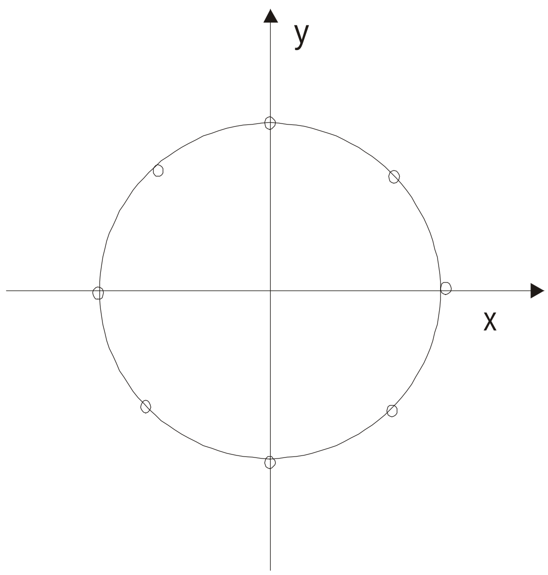
d) Repeat part (b) for the case where we replace the positive charge at x=+d/2 with an equal negative charge –Q. What direction does the field produced by both charges point on the x-axis, y-axis and z-axis? Show the directions below. Also show any locations where the electric field or the potential equal zero and evaluate V at the origin. Sketch 3 typical equipotentials.





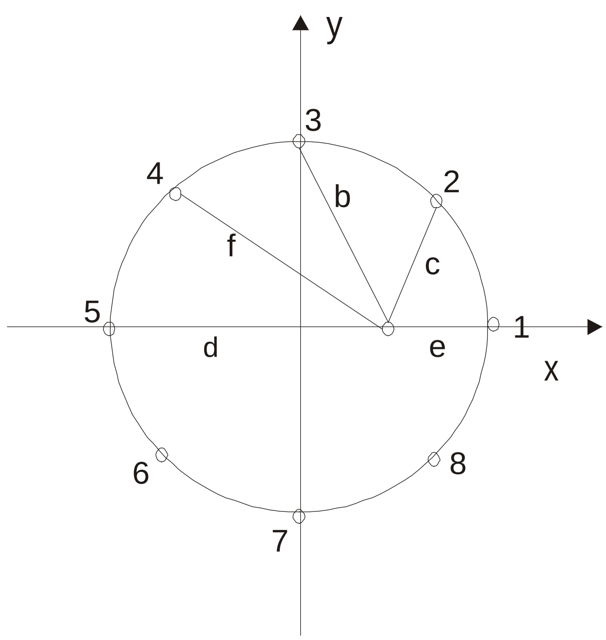
e) Repeat part (c) for the case where we replace the positive line charge at x=+d/2 with an equal negative line charge $-\rho_{Lo.}$ *Note: It is no longer necessary to do this part.*

f) For the final parts of this problem, we will limit ourselves to only the cylindrical case with line charges. First, find the direction of the electric field along the x and y axes for the configuration below where each of the eight dots represents the position of a positive line charge with density ρ_{Lo}. In addition, indicate all the lines of symmetry for this configuration. Assume in this and the next part of this problem that the line charges are located on a cylinder of radius a.



One of the counterintuitive results we can obtain from Gauss' law is the following. If we have either a sphere or a cylinder of uniform charge with a concentric empty space at its center, the electric field in the empty region will be zero. This is discussed on pages II-25 and II-26 of the notes and in examples like 3.12 of the text. As a general rule, if there is sufficient symmetry in the problem to solve it using Gauss' law, then an empty region such as these will have no field in it. To see how this can be, we will use Coulomb's law to show that the electric field inside a cylindrical sheet of charge is zero. This is sufficient to demonstrate that the general principle holds, since any cylindrically symmetric charge distribution can be approximated, with arbitrarily good accuracy, using a series of such charge sheets. Rather than doing this problem analytically, we will approximate the cylindrical sheet of charge with the eight line charges we just looked at above.

(g) Again assuming that the eight line charges are located on a cylinder of radius a, determine the electric field at the location x = a/2, y = 0 for all z, as shown in the figure. The answer to this question would be zero if we used a very large number of line charges to represent the surface charge. Compare your result to the field magnitude you would get if you treated the E field like a scalar and just added the magnitudes of the contributions from the eight line charges. (This part has now been done for you on the next page). Discuss why your answer makes sense. *Note: You can do this problem by adding up the contributions from the eight line charges by hand or you can use Matlab, Maple, or even Excel to do much of the work for you. However, if you use Matlab or Excel you will need to use a number for the line charge density. To keep the analysis simple, assume that the line charge density for each of the eight line charges is equal to 1.0 and then multiply your answer by* ρ_{Lo} *when you are finished.*



h) To complete this analysis, determine the surface charge density ρ_{So} represented by this configuration. Give your answer in terms of ρ_{Lo} and the radius a.

Some additional information on the location of the line charges.

Locations: Line 1 at $(a,0)$, line 2 at $\left(\frac{a}{\sqrt{2}},\frac{a}{\sqrt{2}}\right)$, line 3 at $(0,a)$, line 4 at $\left(-\frac{a}{\sqrt{2}},\frac{a}{\sqrt{2}}\right)$, line 5 at $(-a,0)$. The other three are at symmetric locations. The distances from the lines to the field point at $\left(\frac{a}{2},0\right)$ are labeled in the figure and are b = 1.1a, c = 0.7a, d = 1.5a, e = 0.5a and f = 1.4a. The angle between line c and the x-axis is 73.7°, the angle between line b and the x-axis is 63.4° and the angle between line f and the x-axis is 30.4°.

For reference, adding all eight field magnitudes (treating the electric field as if it were a scalar) gives the following total:

$$E_{Total} = \frac{\rho_{Lo}}{2\pi\varepsilon_o}\left[\frac{1}{0.5a}+\frac{1}{1.5a}+\frac{2}{.7a}+\frac{2}{1.1a}+\frac{2}{1.4a}\right] = \frac{\rho_{Lo}}{2\pi\varepsilon_o}[8.6]$$

Note that the terms with 2 in the numerator are actually two terms with identical contributions, like from lines 4 and 6, 3 and 7, 2 and 8.