Fields and Waves I

Name _______________ ECSE-2100 Spring 2000 Section ____________

Preparation Assignments for Homework #3

Due at the start of class.

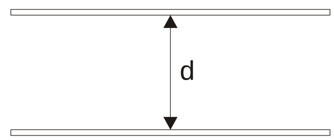
Reading Assignments

Please see the handouts for each lesson for the reading assignments.

3,4 February Lesson 2.5

No prep assignment to do, but here are four questions anyway.

1. A parallel plate capacitor consists of two parallel conducting plates separated by a distance d. The voltage on one plate is V_o and the other plate is grounded. The region between the plates is air. What is the electric field in the region between the plates? What is the electric field when the region between the plates is filled with an insulator with dielectric constant ε ?



2. For the same parallel plate capacitor, what is the potential as a function of position in the region between the plates, both for air and insulator cases?

3. What is the approximate capacitance of the earth, if we assume it is an isolated conductor. Make a reasonable assumption for the radius of the earth if you don't know it.

4. Given that the electric field near the surface of the earth is about 100 V/m, estimate the total amount of charge on the earth. Hint: you should be able to get the charge density from the boundary condition for the electric flux density

Due 7 February (4 points) Lesson 2.6

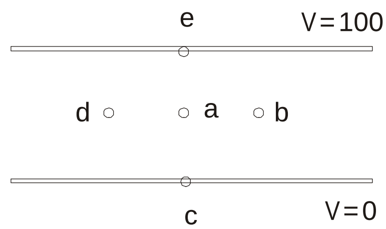
1. Draw a picture of the two-wire transmission line studied in the previous class (lesson 2.5). Indicate on this picture where the charge density is the largest.

2. Write the expression for the energy stored in a capacitor in terms of capacitance C and charge stored Q.

3. Show that the answer to question 2 for the previous class (the voltage as a function of position in a parallel plate capacitor) satisfies Laplace's equation.

4. Shown below is a drawing of a simple parallel plate capacitor. The top plate voltage is 100 volts and the bottom plate is grounded. Determine the voltage at each of the points

marked a, b, c, d, and e. Show that the voltage at point a is equal to the average of the other four voltages. That is, $V(a) = \frac{V(b)+V(c)+V(d)+V(e)}{4}$.



Due 9 February (4 Points) Lesson 2.6

1. The extremely large values of capacitance available these days from capacitors like PowerCache (see http://www.powercache.com/), are possible because of very large areas per gm of capacitor and very small separation distances. To have a capacitance of about 10 Farads with a separation of about 10 Angstroms, how large must the plate area be?

2. Is the voltage $V(x,y) = 5x^2 - 5y^2$ a solution to Laplace's equation?

3. A coaxial cable with inner radius a = 1 mm and outer radius b = 5 mm has a voltage of 10 V on the inner conductor and the outer conductor is grounded. Assume that the insulator between the conductors is air. Evaluate the electric field at the outer conductor using the exact expression for the field found in example 3.20. Now evaluate the field with a very crude approximation by taking the difference between the outer and inner potential and dividing by the distance between them. How do the two numbers compare?

4. Write the finite difference version of Laplace's equation.

Class time 10,11 February

Open shop to work on Homework 3. Due at 5 pm on 11 February.

Due 14 February (4 Points) 3.1

1. It is typical to limit the current in a household circuit to 10-20 amps. What is the current density in a 12 gauge cable carrying a current of 10 amps? Check the supplemental information webpage for cable information.

2. Determine as much practical information as you can about the RG/58U coaxial cable we use in the studio.

3. What is the resistance of a 100 meter long copper wire with a diameter of 2mm?

4. What is the resistance of a 100 meter long wire with a diameter of 2mm if the wire is made from hard rubber (see textbook).

Homework #3

Problem 1 – (10 points)

There are a small number of simple conductor/dielectric configurations for which we can relatively easily find the capacitance. Students of electromagnetics should be sure that they know how to derive and use all of the expressions for such capacitors. The list of simple capacitors should include the following:

1. Spherical Capacitor – see example 3.21
2. Spherical Capacitor with more than one dielectric – problem 3.9.8
3. Two-wire transmission line – see examples 3.19, 3.25, 3.26 for increasing accuracy
4. Coaxial Cable – see example 3.20 and problem 3.9.2
5. Coaxial Cable with more than one dielectric – problems 3.9.3 and 3.9.7
6. Parallel Plate Capacitor – see example 3.17 and equation (111)
7. Parallel Plate Capacitor only partially filled with dielectric or with more than one dielectric – problems 3.9.1, 3.9.4, 3.9.5
8. Parallel plate transmission line – see section 7.1.1

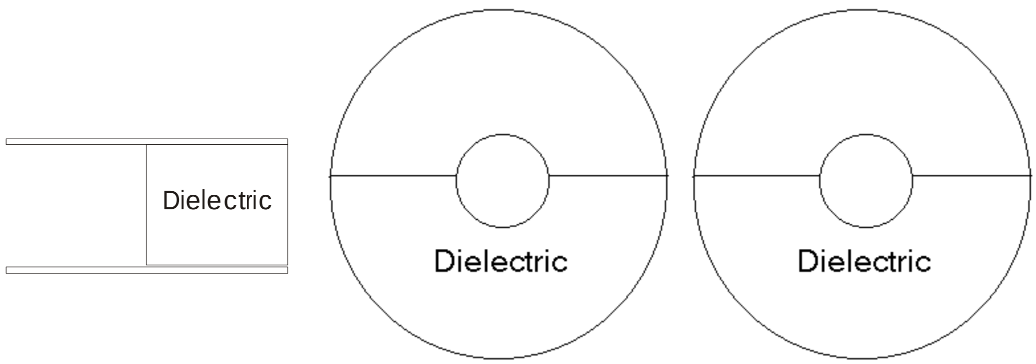
In this problem, we want to look at some variations of these standard configurations. We will consider three standard capacitors: parallel plate, cylindrical (coaxial) and spherical. Look at the analysis of the parallel plate capacitor in example 3.17, the coaxial cable in example 3.20 and the spherical capacitor in example 3.21 and answer the following questions. (You also analyzed the field structure for the spherical capacitor in the first quiz.)

a. Assume that there is a grounded planar conductor at $x = 0$ and a planar conductor at $x = d$ with a voltage V_o. Write the electric field in the region between the plates ($0 < x < d$) in terms of V_o instead of in terms of the charge on the plates Q. Write your answer for both the case where there is no dilectric between the conductors ($\varepsilon = \varepsilon_o$) and where there is a dielectric between the conductors ($\varepsilon = \varepsilon_o$). Note that the two expressions should be the same.

b. Assume that there is a grounded cylindrical conductor at $r = b$ and a conductor at $r = a$ with a voltage V_o. Write the electric field in the region between the plates ($a < r < b$) in terms of V_o instead of in terms of the charge Q. Write your answer for both the case where there is no dielectric between the conductors ($\varepsilon = \varepsilon_o$) and where there is a dielectric between the conductors ($\varepsilon = \varepsilon_o$) Note that the two expressions should be the same.

c. Assume that there is a grounded spherical conductor at r = b and a conductor at r = a with a voltage V_o. Write the electric field in the region between the plates (a < r < b) in terms of V_o instead of in terms of the charge Q. Write your answer for both the case where there is no dielectric between the conductors ($\varepsilon = \varepsilon_o$) and where there is a dielectric between the conductors ($\varepsilon = \varepsilon_o$) Note that the two expressions should be the same.

Now, assume that, for each of these configurations, half of the space between the conductors is filled with the dielectric and half is empty (air), as shown below.



d. Using your expressions for the electric field, show that the appropriate boundary conditions are satisfied at the interface between the dielectric and empty regions.

e. Using the boundary condition for the normal component of D at a conductor-dielectric interface (you will have to first find D from your expressions for E), determine the charge density and then the total charge on the grounded conductor in each case.

f. From the total charge for each case and the voltage difference between the two conductors, determine the capacitance.

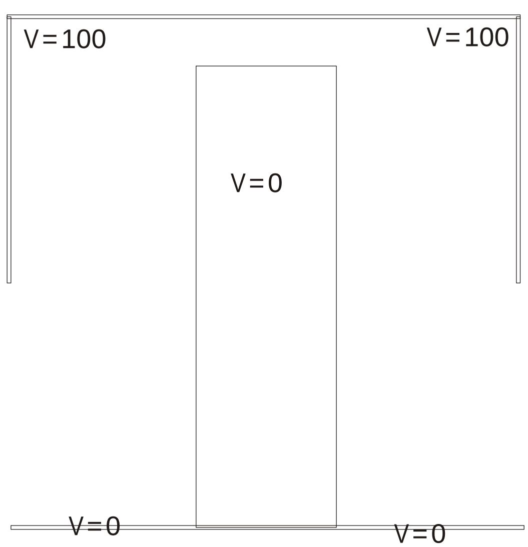
Problem 2 – (10 points)

Nearly all practical electromagnetics problems are analyzed using numerical methods rather than the analytical methods we have been addressing. One of the most common techniques is called the Finite Difference Method, which we have considered in class. In this problem, we will try to find the capacitance of a simple two-dimensional problem with a somewhat odd geometry. To address this geometry, we will use a spreadsheet to solve Laplace's equation. Review the materials on numerical methods in the class notes and on reserve in the library.

The method is reasonably simple. First, identify the cells you will use to represent the conductors. Set the value in these cells equal to the potential on the conductor. Second, all exterior cells must have either a specific potential or be set equal to their nearest interior neighbor. This is the equivalent to setting the normal derivative of the potential equal to zero. This boundary condition is quite accurate if the boundary in question is a line of symmetry for the problem. As we will see, this will be the case in this problem. Third, for all interior points, set the voltage in each cell equal to the average of its four nearest neighbors. This is the finite difference equivalent of Laplace's equation. After you enable iteration, the spreadsheet values will eventually converge to something near the correct voltages at each location. Use this information to determine the capacitance per unit length of this configuration, following the method we discussed in class. For your solution, use at least a 25 x 25 array. Whatever information you use from your spreadsheet has to be included with your analysis.

The geometry you are to consider is shown below. Assume that it continues for a very long distance into the page, just like a coaxial cable. Thus, you will be finding the capacitance per unit length, just as is the case with the cable. Also, assume that the structure is periodic in the horizontal direction. That is, it repeats over and over. That means that the left and right boundaries are lines of symmetry, so that for the open parts of the boundary, the normal derivative boundary condition is the appropriate choice. Wherever there is a conductor at the boundary, the voltage on the conductor should be the boundary condition. It is easier to address this problem if we assume that this region is 24mm by 24mm and the conducting pedestal 20mm by 4mm and in the exact center of the bottom. Assume that the top electrode is at 100 volts and the bottom is grounded.

To summarize – solve for the potential at all points represented by cells and use this information to determine the capacitance per unit length of this structure. Discuss why you think your answer is at least approximately correct.



Extra Credit – Fill the upper half of the open region with Teflon. That is, everything from the bottom edge of the upper conductor to the top of the structure. Determine the capacitance for this more complex configuration and discuss the accuracy of your answer.