Lesson Summaries

Version 1.4

For each of the in-class lessons, a summary of what should have been learned is presented. The particular problems and experiments done in each lesson contain examples of each item listed. Other examples are included in the text and class notes.

Lesson 1 – Intro to Fields and Vector Mathematics Review

In these lessons, the basic vector mathematics necessary to apply Maxwell's Equations and the related equations (e.g. Coulomb's Law and the Biot-Savart Law) is investigated. As a general rule, one need only look at Maxwell's Equations in both integral and differential form to see the mathematical tools we require in this course.

1.1 Intro to Fields and Waves

- In a circuit application, capacitive effects can occur due to proximity of wiring, the box enclosing the circuit, etc.
- In a circuit application, inductive effects can occur when wire loops surround the same region of space. A transformer, even if it is a poor one, can be created by wrapping a few turns of wire around a conventional solenoidal inductor.
- Wires, whether open or closed loops can pick up a large variety of noise signals from standard electrical devices such as power cords, computer screens etc.
- Standard cabling, such as a coaxial cable, behaves very differently at low and high frequencies. Low frequencies are those for which the physical dimensions of the system of interest are very small compared to a wavelength.
- Transmission lines must be terminated properly to work correctly.

1.2 Basic Math and Coordinate Systems

- There are three coordinate systems we work with in an introductory level Electromagnetics course – rectangular, cylindrical and spherical
- *In these coordinate systems*, you should be able to:
  - Add and subtract vectors and do dot and cross products
  - Sketch surfaces on which one coordinate is a constant.
  - Find the area of surfaces on which one coordinate is a constant
  - Identify the vector surface element $d\vec{S}$ for each surface in item 2.
  - Sketch volumes defined by coordinate surfaces (surfaces on which a coordinate is a constant).
  - Find the volumes of item 5.
  - Integrate functions over volumes of the form $\int f(\vec{r})dv$, where $f(\vec{r})$ is some function expressed in one of the three standard coordinate systems.
- You should know the surface areas and volumes of rectangles, cylinders and spheres.

- You should know the line elements, surface elements and volume elements for each system.
- You should be able to convert a vector from one coordinate system to another.

1.3 Gradient, Line Integrals and Curl

You should be able to:

- Do closed line integrals of the form $\oint \vec{B} \cdot d\vec{l}$, particularly for paths on which a coordinate is a constant.
- Take the curl $\nabla \times \vec{B}$ of vector functions we will find as electric and magnetic fields. These usually depend on a single coordinate and have only one component, but not in every case.
- Take the gradient of a scalar ∇f to obtain a vector function

In addition, you should understand:

- Stokes Theorem and how to apply it
- That fields with no curl satisfy $\oint \vec{B} \cdot d\vec{l} = 0$
- That fields with no curl can be represented by the gradient of a scalar.

1.4 Surface Integrals and Divergence

You should be able to:

- Evaluate vector area integrals of the form $\int \vec{A} \cdot d\vec{S}$ for surfaces on which a coordinate is a constant.
- Evaluate the divergence $\nabla \cdot \vec{A}$ of a vector.

In addition, you should understand:

- The divergence theorem and how to apply it.

1.5 Wave Properties and Phasors

Will be added in a future version of this list.

Lesson 2 – Electric Fields

In these lessons we learn how to apply Coulomb's Law, Gauss' Law and Poisson's Equation to find the electric field $E(\vec{r})$ and the electric potential $V(\vec{r})$ from a given charge distribution with and without conductors and dielectrics. The methods learned can mostly only be applied to highly symmetric problems (describable with a single coordinate). The exception is direct application of Coulomb's Law for a very limited set of problems and the application of a simple finite difference numerical technique. We also address the boundary conditions for electric fields and electric flux density $\vec{D}(\vec{r})$, particularly at boundaries between two insulators or between an insulator (also known as a dielectric) and a conductor. Finally, we address how to determine the capacitance and the stored energy of a conductor/insulator configuration.

2.1 Coulomb's Law and Charge

You should be able to:

- Find the total charge in some volume defined by coordinate surfaces.
- Find the total charge on some surface on which a coordinate is constant
- Find the total charge on some line
- Find the total charge in some region if it consists of a series of point charges
- Sketch the electric field lines for a small number of point charges (both positive and negative). A small number is usually something less than 10.
- Determine the force on a charge due to an electric field.

2.2 Gauss' Law

You should be able to:

- Show that the electric field of a point charge satisfies Gauss' Law in integral form.
- Identify the symmetries of simple charge distributions
- Determine the direction of the electric field from the symmetries of the charge distribution
- Determine the surfaces over which the electric field is a constant or is not in the direction of the surface and, thus, $\int \vec{E} \cdot d\vec{S} = 0$, for surfaces described simply in one of the three standard coordinate systems.
- Sketch a Gaussian surface (one that can be used with Gauss' Law in integral form to find the electric field).
- Use Gauss' Law in integral form to find the electric field of charge distributions that are symmetric in one of the standard coordinate systems.
- Verify that you have the correct solution for the electric field by applying Gauss' Law in differential form and showing that the electric field has no curl.

2.3 Electric Potential (Voltage)

You should be able to:

- Determine the electric potential at some point in space using a given expression for the electric field. The result in this case should be a number, not a function.
- Determine the electric potential function (a scalar field) in some region using a given expression for the electric field.
- Verify that your expression for the potential function is correct by evaluating $-\nabla V(\vec{r})$.
- Sketch electric field lines and equipotential lines (actually surfaces, but they look like lines in two dimensions) for a small number of positive and negative charges.

- Determine electric potential function from a given charge distribution using Coulomb's law.
- Determine the electric field either directly from the charge distribution or from an expression for the potential. When finding the electric field directly from Coulomb's law, only simple, highly symmetric configurations will be considered.

2.4 Electric Materials – Conductors and Dielectrics

You should be able to:

- Identify the direction of the electric field if given a set of equipotentials
- Sketch equipotentials and electric field lines for a simple set of conducting electrodes.
- Roughly sketch the surface charge density on the surface of a conductor
- Apply Gauss' law to a combination of conductors and insulators that can be described by a single coordinate in any of the three standard coordinate systems.
- Determine where the charges reside in a configuration that can be analyzed using Gauss' law.
- Identify where the electric field in an insulator will exceed the dielectric strength of the materials and thus where breakdown will occur.
- Apply the boundary conditions for the electric field and the electric flux density at the surfaces of materials (conductors and dielectrics) found in problems that can be analyzed using Gauss' law. (In these cases the electric field and the electric flux density will either be normal or tangent to the surface.)
- Apply the boundary conditions for the electric field and the electric flux density at a dielectric—conductor or a dielectric—dielectric boundary when the electric field is neither normal or tangent to the surface.

2.5 Capacitance & Energy

You should be able to:

- Find the capacitance of a standard electrode configuration using $Q = CV$.
- Find the stored energy in a capacitor by integrating the energy density over the system volume.
- Find the capacitance using the stored energy and $W_m = \frac{1}{2}CV^2$.
- Build and test a small capacitor using two parallel wires on a protoboard. Compare the measured capacitance value with the calculated capacitance for this system.
- Fully analyze the capacitance and energy storage of two standard cable configurations – coaxial and two-wire (usually implemented as twisted pairs).

2.6 Laplace and Poisson's Equations

You should be able to:

- Find the analytical solution to one-dimensional electric field problems using Laplace's or Poisson's equations. (These are the same problems we can solve using Gauss' Law.)
- Write the finite difference versions of Laplace's and Poisson's equations.
- Solve the finite difference Laplace's equation using a spreadsheet . Cases are limited to two-dimensional problems. However, both open and closed boundaries and problems with and without dielectric materials that partially fill the field regions must be considered.

2.7 Method of Images *(Note: We did not cover this material this term.)*

You should be able to:

- Solve for the electric potential V and electric field E of either a point charge or a line charge located near a conducting plane. In the case of the line charge the line should be parallel to the plane.
- Use the results of this analysis to find the capacitance of wires over ground planes or charged spheres over ground planes.

Lesson 3 – Magnetic Fields

In these lessons we learn how to apply Ampere's Law and the Biot/Savart Law to find the magnetic field $\vec{B}(\vec{r})$ and the magnetic vector potential $\vec{A}(\vec{r})$ from a given current distribution with and without conductors and magnetic materials. The methods learned can mostly only be applied to highly symmetric problems (describable with a single coordinate). The Biot/Savart Law will be used primarily to address the symmetries of the problems. We will also learn to apply Ohm's Law in point form to find the resistance of conductors. We also address the boundary conditions for the magnetic flux density $\vec{B}(\vec{r})$ and the magnetic field intensity $\vec{H}(\vec{r})$, particularly at boundaries between two magnetic materials or between a magnetic material and a conductor. We address the application of Faraday's Law to find the voltage induced in a circuit due to a changing magnetic flux. We address how to determine the mutual and self inductance, the stored energy of and forces exerted on a conductor/magnetic material configuration. Finally, we learn a little bit about permanent magnets and the magnetic circuit analysis technique.

3.1 Currents and Resistance

You should be able to:

- Find the total current carried by a conductor from the current density.
- Determine the resistance of a conductor with a uniform current distribution
- Determine the resistance of a conductor with a non-uniform current distribution
- Measure the resistance of a cable and compare with calculations

3.2 Magnetic Fields and Ampere's Law

You should be able to:

- Determine from a field line plot or from a field expression whether a given field could or could not be a magnetic field.
- Determine the symmetries of standard geometries such as coaxial cables, solenoids and toroids.
- Determine the magnetic field (either $\vec{B}$ or $\vec{H}$) from a given current density for any of the simple one-dimensional standard geometries.

3.3 Flux and Magnetic Vector Potential

You should be able to:

- Interpret a flux diagram (a collection of typical magnetic field lines) to determine, for example, the flux at one location in terms of the flux at another.
- Determine the internal or external flux produced by a given current distribution using either the magnetic field $\vec{B}(\vec{r})$ or the magnetic vector potential $\vec{A}(\vec{r})$. That is, by apply either $y_m = \int \vec{B} \cdot d\vec{S}$ or $y_m = \oint \vec{A} \cdot d\vec{l}$.

3.4 Faraday's Law

You should be able to:

- Find the magnetic field (either $\vec{B}$ or $\vec{H}$) from a given standard current distribution
- Find the flux in a typical coil cross-section using either $y_m = \int \vec{B} \cdot d\vec{S}$ or $y_m = \oint \vec{A} \cdot d\vec{l}$.
- Find the total flux linked by a coil $\Lambda = Ny_m$
- Determine the emf $V(t) = -\frac{d\Lambda}{dt}$ induced in a coil by a changed flux linkage
- Perform a simple experiment that shows the flux from one circuit linking another
- Find the emf produced in a loop moving through a static magnetic field.

3.5 Inductance

You should be able to:

- Find the self-inductance of a simple configuration such as a coaxial cable, a solenoid or a torus using the expression $L = \frac{\Lambda}{I}$ after first finding Λ following the procedures of lesson 3.4.
- Find the mutual-inductance of two simple configurations located in reasonably close proximity to one another.

- Find the approximate self or mutual inductance of a simple circuit using information provided on the magnetic field at some typical points. This information could either have been determined analytically or using numerical methods.
- Perform an experiment to show that the calculation of the mutual inductance is in reasonable agreement with actual application

3.6 Magnetic Materials

You should be able to:

- Calculate the magnetic field (either $\vec{B}$ or $\vec{H}$) from a given standard current distribution including the effects of magnetic materials.
- Determine the change in direction of a magnetic field at the boundary between two magnetic materials
- Perform an experiment that shows the ability of a magnetic material to conduct magnetic flux from one coil to another and thus improve coupling between the coils

3.7 Magnetic Circuits

You should be able to:

- Apply the magnetic circuit method to determine the magnetic field and flux in a configuration of coils and magnetic materials.
- Use the magnetic field information obtained using the magnetic circuit method to determine inductance and energy storage of a simple configuration

3.8 Magnetic Energy and Force

You should be able to:

- Determine the energy stored in a magnetic field configuration
- Find the inductance from the energy stored in a magnetic field configuration
- Qualitatively describe the force between a magnetic field and a magnetic material (for example between a permanent magnet and a piece of iron). That is, estimate changes in energy as the magnetic field and the magnetic material move with respect to one another.

Lesson 4 – Transmission Lines

In these lessons, we address what happens to voltages and currents on transmission lines (cables at sufficiently high frequency that wave phenomena become important). More class time is spent doing experiments than in any other part of the course. We study quite thoroughly the properties of 100 meter lengths of RG58A/U coaxial cable, since that is the cable we use to connect the instruments in the studio classroom. Also, this is very much a standard cable with many, many uses in the electrical and electronics industry. For example, the function generator we use (the HP 33120A) assumes it is driving a 50 ohm load since it is almost always connected to a cable like the RG58A/U. All cables are characterized by a capacitance, inductance, resistance and conductance per unit length and can be approximated by an artificial transmission line constructed from a finite number of lumped circuit components (usually just inductors and capacitors). Such an artificial line with 20 inductors and capacitors is used to study what happens as the voltage and current waves move from one end of the cable to the other. We begin by looking at the propagation of sinusoidal voltage and current waves down such a cable and develop some simple phasor expressions for V(z) and I(z). Phasor notation is very useful here, since it greatly simplifies the mathematics. Thus, it is essential to review the basics of complex numbers. The form of the solutions observed for transmission lines turns out to be identical to all functions that are characterized by a wave equation. This is useful to keep in mind when we get to electromagnetic waves (lesson 5). The form of the solution remains the same, regardless of the problem considered. Thus, the first step in almost every wave problem is to write down the general form of the answer and then the solution method allows us to evaluate the parameters in the solution. After studying a variety of phenomena we see with both lossless and lossy transmission lines, we finally consider the propagation of pulses.

4.1 Introduction to Transmission Lines

You should observe

- It takes a finite amount of time for a voltage wave to propagate from one end of a cable to the other.
- The delay time between the input and output of a transmission line is consistent with the velocity of propagation on the line.
- The voltage and current on a transmission line are related by the characteristic impedance.

You should be able to

- Determine the inductance and capacitance per unit length for a cable and determine the values of lumped circuit elements that model a particular length of cable.
- Find the propagation velocity from the inductance and capacitance per unit length.
- Determine the characteristic impedance for a transmission line.

4.2 Pulses and Transients on Transmission Lines (Now done after 4.6)

You should observe

- Observe that it takes a discrete pulse a finite amount of time to propagate from the input to the output of a cable.
- The input pulse reappears at the input (with the appropriate delay) if the load is not matched to the line.
- The pulse returns with a positive magnitude if the load impedance is larger than the characteristic impedance and a negative magnitude if the load impedance is smaller than the characteristic impedance.

You should be able to

- Determine the amplitude of the initial pulse launched on a transmission line using the voltage divider relationship with the input impedance of the line equal to the characteristic impedance.
- Determine the time it takes for a pulse to propagate from the input to the output end of the cable using the propagation velocity.
- Determine the unknown length of a cable from a given propagation time and the propagation velocity for the cable.
- Determine the reflection coefficients at the load and the source.
- Make a lattice diagram for several transits of the cable by an input pulse.
- Determine the voltage at the input as a function of time.
- Determine the voltage at the load as a function of time.
- Determine the voltage at any location on the line as a function of time.

4.3 Standing Wave Patterns

You should observe

- The voltages observed at the various nodes of the lumped (artificial) transmission line vary significantly with position when the load impedance is not equal to the characteristic impedance. When the two impedances are equal, the voltages at the various nodes will be about the same, possibly becoming somewhat smaller nearer the load. The voltage maxima and minima will have roughly the same values and will be separated by a constant distance. Two maximas or two minimas will be separated by a half wavelength.
- For load impedances that are real and greater than the characteristic impedance, the voltage at the load will be a maximum, while it will be a minimum for real load impedances that are smaller than the characteristic impedances.
- When the load is either capacitive or inductive, the voltage at the load will be neither an maximum nor a minimum.
- The input impedance of a shorted transmission line depends on frequency.

You should be able to

- Determine the propagation constant b and the wavelength l for a voltage wave at a given frequency.
- Determine the reflection coefficient at the load and the voltage standing wave ratio.
- Sketch voltage and current standing wave patterns for lossless lines with either a real resistive load or an inductive or capacitive load. (Knowing how to address real resistive loads is more important that general complex loads.)

4.4 Sinusoidal Circuits on Transmission Lines

You should observe

- The input impedance of a transmission line with a matched load will equal the characteristic impedance of the line. For any other load, whether real or complex, the input impedance of the line will be complex.
- Under certain conditions the input impedance of a lossless line can look either inductive, capacitive or resistive.

You should be able to

- Calculate the input impedance of a lossless transmission line.
- Sketch the real and imaginary parts of the input impedance as a function of frequency.
- Determine the voltage at the input and at the load.
- Determine the voltage in phasor form as a function of z everywhere on the transmission line.
- Experimentally determine the input impedance of a transmission line for a matched load and real resistive loads that are either larger or smaller than the characteristic impedance. (In this lesson we also address capacitive and inductive loads, but we did not analyze them in detail.)

4.5 Lossy Transmission Lines

You should be able to

- Determine the skin depth in typical metals at specified frequencies.
- Determine the resistance per unit length and the conductance per unit length for standard cables (coax, stripline, etc.).
- Determine the propagation constant $g = a + jb$ and the characteristic impedance of a low-loss line.

4.6 Transmission Line Matching and Smith Charts

You should be able to

- Find the reflection coefficient for a complex load.
- Determine the standing wave ratio for a lossless line with a complex load impedance.
- Connect two transmission lines in parallel or connect a transmission line in parallel with a load impedance and be able to determine the net impedance produced by the combination.
- Find the new reflection coefficient and voltage standing wave ratio for the combination load impedance.
- Repeat the tasks above using a Smith Chart. (This is basically optional, since we did not spend any real time on the Smith Chart.)